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Propagation dynamics of reaction-diffusion equations in a time-heterogeneous shifting environment

In this paper, we study the propagation dynamics of a large class of time and space heterogeneous reaction-diffusion equations

$$u_t = u_{xx} + ug(x - \omega(t), t, u), \quad x \in \mathbb{R},$$

where $\omega(t)$ represents the shifting distance, and the nonlinearity $ug(\xi, t, u)$ is asymptotically of KPP type as $\xi \rightarrow -\infty$ and is negative as $\xi \rightarrow +\infty$. Let c^* be the spreading speed of the limiting equation $u_t = u_{xx} + ug(-\infty, t, u)$. Under the assumption that the shifting speed $\omega'(t)$ admits a uniform mean c , we show that the solutions with compactly supported initial data go to zero eventually when $c \leq -c^*$, the leftward spreading speed is $-c^*$ when $c > -c^*$, and the rightward spreading speed is c and c^* when $c \in (-c^*, c^*)$ and $c \geq c^*$, respectively. We also establish the existence, uniqueness and nonexistence of the forced traveling wave in terms of the sign of $c - c^*$. This talk is based on a joint work with Prof. Xiao-Qiang Zhao.