DIMITRIS KOUKOULOPOULOS, University of Montreal
Rational approximations of irrational numbers

Given quantities $\Delta_1, \Delta_2, \dots \geqslant 0$, a fundamental problem in Diophantine approximation is to understand which irrational numbers x have infinitely many reduced rational approximations a/q such that $|x - a/q| < \Delta_q$. Depending on the choice of Δ_q and of x, this question may be very hard. However, Duffin and Schaeffer conjectured in 1941 that if we assume a "metric" point of view, the question is governed by a simple zero–one law: writing φ for Euler's totient function, we either have $\sum_{q=1}^{\infty} \varphi(q)\Delta_q = \infty$ and then almost all irrational numbers (in the Lebesgue sense) are approximable, or $\sum_{q=1}^{\infty} \varphi(q)\Delta_q < \infty$ and almost no irrationals are approximable. In this talk, I will present the history of the Duffin–Schaeffer conjecture and the main ideas behind the recent work in collaboration with James Maynard that settled it.