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Hermitian self-orthogonal codes

Let C be a $[n,k]_{q^2}$ linear code,.

C is linearly equivalent to a Hermitian self-orthogonal code if and only if there are non-zero $\lambda_i \in \mathbb{F}_q$ such that

$$\sum_{i=1}^{n} \lambda_i u_i v_i^q = 0,$$

for all $u, v \in C$.

For any linear code C over $\mathbb{F}_{q^2}$ of length n, Rains defined the *puncture code* $P(C)$ to be

$$P(C) = \{\lambda = (\lambda_1, \dots, \lambda_n) \in \mathbb{F}_q^n \mid \sum_{i=1}^{n} \lambda_i u_i v_i^q = 0, \text{ for all } u, v \in C\}.$$

C has a truncation of length $r \leqslant n$ which is linearly equivalent to a Hermitian self-orthogonal code if and only if there is an element of $P(C)$ of weight r.

Rains was motivated to look for Hermitian self-orthogonal codes, since there is a simple way to construct a $[\![n, n-2k]\!]_q$ quantum code, given a Hermitian self-orthogonal code.

In this talk, I will detail an effective way to calculate the puncture code. I will outline how to prove various results about when a linear code has a truncation which is linearly equivalent to a Hermitian self-orthogonal linear code and how to extend it to one that does in the case that it has no such truncation. In the case that the code is a Reed-Solomon code, it turns out that the existence of such a truncation of length r is equivalent to the existence of a polynomial $g(X) \in \mathbb{F}_{q^2}[X]$ of degree at most $(q-k)q-1$ with the property that $g(X) + g(X)^q$ has $q^2 - r$ distinct zeros in $\mathbb{F}_{q^2}$.