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Sampling issues for the X-ray transform on simple surfaces

Abstract: On a Riemannian manifold-with-boundary, the geodesic X-ray transform maps a function to the collection of its integrals over all geodesics through the domain, with applications to medical imaging and seismology. In the literature, it is now well-known that injectivity, stability (or mild instability) and inversion formulas hold at the continuous level, for example when (M, g) is a 'simple' surface. By 'simple' we mean (i) no infinite-length geodesic, (ii) no conjugate points, (iii) strictly geodesically convex boundary, arguably the most inversion-friendly case.

In this talk, I will discuss the issue of proper discretizing and sampling of the X-ray transform, addressing the following: (a) Given a bandlimited function f, what are the minimal sampling rates needed on its X-ray transform $I_0 f$ for a faithful (=unaliased) recovery? (b) In the case where data is sampled below the expected requirements, can one predict the location, orientation and frequency of the artifacts generated?

The main tools to answer (a)-(b) are a combination of a reinterpretation of the classical Shannon-Nyquist theorem in semi-classical terms, as initiated in [P. Stefanov, https://doi.org/10.1137/19M123868X], and an accurate description of the canonical relation of the X-ray transform viewed as a (classical, then semi-classical) Fourier Integral Operator. The answer also depends on geometric parameters of the surface (e.g., curvature and boundary curvature), and on the coordinate system used to represent the space of geodesics. Several (unaliased and aliased) examples will be given throughout.

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