
ILYA AMBURG, PNNL

Localization in harmonic vectors of simplicial complexes

Simplicial complexes allow for modeling higher-order interactions beyond the standard pairwise graph paradigm, and naturally lend themselves to analysis of edge flows through the spectra of associated higher-order Laplacians. However, theoretical foundations and principled tools to justify and unify the various methods that have been introduced to study harmonic flows are sorely missing. In particular, at the heart of many edge signal analysis algorithms lie implicit assumptions regarding localization of harmonic vectors around “holes” in the complex. Here, we begin the principled study of localization properties of harmonic vectors. We provide theoretical insights for why localization of harmonics is expected given some assumptions about the underlying simplicial complex, and demonstrate that it occurs in empirical networks from a wide range of disciplines. We leverage these results to construct a hole localization algorithm that performs many orders of magnitude faster than state-of-the-art algorithms while yielding solutions with more desirable properties. The resulting localized harmonics also provide rich features, which we demonstrate increase performance in various machine learning tasks.