Combinatorics and geometry of moduli of curves
Combinatoire et géométrie des modules de courbes

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ADAM AFANDI, WWU Münster

Combinatorial Structures of Hyperelliptic Hodge Integrals

A hyperelliptic Hodge integral is a type of intersection number on the moduli space of hyperelliptic curves. In the past, various mathematicians have discovered that these intersection numbers, when restricted to various families, tend to have remarkable combinatorial structure. However, in general, these intersection numbers are difficult to compute, and it is unclear what combinatorial structure governs all of them. In this talk, I will present some results that allow us to compute hyperelliptic Hodge integrals using elementary symmetric functions. At the end, I will talk about some conjectures and open problems.

SAMIR CANNING, University of California San Diego

The Chow rings of moduli spaces of elliptic surfaces

For each nonnegative integer N, Miranda constructed a coarse moduli space of elliptic surfaces with section over the projective line with fundamental invariant N. I will explain how to compute the Chow rings of these moduli spaces when $N \geq 2$. The Chow rings exhibit many properties analogous to those expected for the tautological ring of the moduli space of curves: they satisfy analogues of Faber's conjectures, which in the $N = 2$ case confirms a conjecture of Oprea and Pandharipande on moduli spaces of K3 surfaces polarized by a hyperbolic lattice. Faber's conjectures have led to deep connections between combinatorics and moduli theory, and I will discuss the potential for new connections in the moduli of K3 surfaces setting. This is joint work with Bochao Kong.

FEDERICO CASTILLO, Pontificia Universidad Catolica

When are multidegrees positive?

The multidegrees of X are the analogues of the notion of degree in the multiprojective setting and are fundamental invariants that describe algebraic and geometric properties of X. In this talk we present recent results that provide necessary and sufficient conditions for the positivity of the multidegrees of X. As a consequence of our methods, we show that when X is irreducible, the support of multidegrees forms a discrete algebraic polymatroid.

As an application we will see how this resolves a conjecture by Monical-Tokcan-Yong that Double Schubert polynomials have the saturated Newton property. This is joint work with Yairon Cid-Ruiz, Binglin Li, Fatemeh Mohammadi, Jonathan Montaño and Naizhen Zhang.

EMILY CLADER, San Francisco State University

Permutohedral Complexes and Rational Curves With Cyclic Action

Although the moduli space of genus-zero curves is not a toric variety, it shares an intriguing amount of the combinatorial structure that a toric variety would enjoy. In fact, by adjusting the moduli problem slightly, one finds a moduli space that is indeed toric, known as Losev-Manin space. The associated polytope is the permutohedron, which also encodes the group-theoretic structure of the symmetric group. Batyrev and Blume generalized this story by constructing a "type-B" version of Losev-Manin space, whose associated polytope is a signed permutohedron that relates to the group of signed permutations. In joint work with C. Damiolini, D. Huang, S. Li, and R. Ramadas, we carry out the next stage of generalization, defining a family of moduli space of rational curves with $\mathbb{Z}_r$ action encoded by an associated "permutohedral complex" for a more general complex reflection group, which specializes when $r = 2$ to Batyrev and Blume's moduli space.

CHIARA DAMIOLINI, University of Pennsylvania
Conformal blocks from vertex algebras on $\overline{\mathcal{M}}_{g,n}$

In this talk we will see how representations of Lie algebras or, more generally, modules for a *nice* vertex operator algebra, can be used to define sheaves of conformal blocks on moduli spaces of curves. Under certain assumptions, which I will briefly discuss, these sheaves are locally free and we can explicitly compute their Chern classes via Cohomological Field Theory. This is based on joint work with A. Gibney and N. Tarasca.

ANDY FRY, Pacific University
Moduli Spaces of Rational Graphically Stable Curves

In this talk, we use a graph to define a new stability condition for the algebraic and tropical moduli spaces of rational curves. We will also discuss characterization results in both settings: Tropically, we characterize when the moduli spaces have the structure of a balanced fan by proving a combinatorial bijection between rational graphically stable tropical curves and chains of flats of a graphic matroid. Algebraically, we characterize when the tropical compactification of the moduli space agrees with the theory of geometric tropicalization. Both results occur only when the graph is complete multipartite.

JOSÉ GONZÁLEZ, University of California, Riverside
The Fulton-MacPherson compactification is not a Mori dream space

In 1994, Fulton and MacPherson constructed a compactification $X[n]$ of the configuration space of n distinct labeled points in an arbitrary smooth variety X, which enjoys several desirable properties. To list a few, $X[n]$ is smooth with normal crossings boundary, it has an explicit blowup construction and its geometric points can be given a tree-like description resembling the one of $\overline{M}_{0,n}$. In this talk we show that the Fulton-MacPherson compactification of the configuration space of n distinct labeled points in certain varieties of arbitrary dimension d, including projective space, is not a Mori dream space for n greater than or equal to $d+9$. This is joint work with Patricio Gallardo and Evangelos Routis.

SEAN GRIFFIN, University of California, Davis
Slide rules and tournaments for ω *and* ψ *class products on* $\overline{M}_{0,n}$

We give a positive expansion for the product of any number of ω and ψ classes on $\overline{M}_{0,n}$ in terms of boundary strata using a combinatorial algorithm we call *slide labelings* of trees. We obtain these expansions by constructing a flat family of subschemes whose general fiber is a complete intersection representing the product, and whose special fiber is a generically reduced union of boundary strata. We then give two new combinatorial interpretations of the multidegrees of the embeddings corresponding to ω and ψ classes, one in terms of slide labelings, and one in terms of *lazy tournaments*. This is joint work with Maria Gillespie and Jake Levinson.

IVA HALACHEVA, Northeastern University
Restricting Schubert classes, puzzles, and Lagrangian correspondences

The inclusion of the symplectic Grassmannian of isotropic k-planes into the Grassmannian of all k-planes prompts the question of understanding the pullback map in (equivariant) cohomology in terms of the bases of Schubert classes. We compute this expansion positively in the combinatorial setting of puzzles, by studying a certain duality on them. I will outline this result and describe how a further generalization and geometric interpretation via Lagrangian correspondences can be obtained by upgrading the Grassmannians to their cotangent bundles and the Schubert classes to Segre-Schwartz-MacPherson classes. This is joint work with Allen Knutson and Paul Zinn-Justin.

STEVEN KARP, LaCIM, Université du Québec à Montréal
Wronskians, total positivity, and real Schubert calculus

The totally positive flag variety is the subset of the complete flag variety Fl(n) where all Plücker coordinates are positive. By viewing a complete flag as a sequence of subspaces of polynomials of degree at most n-1, we can associate a sequence of Wronskian polynomials to it. I will present a new characterization of the totally positive flag variety in terms of Wronskians, and explain how it sheds light on conjectures in the real Schubert calculus of Grassmannians. In particular, a conjecture of Eremenko (2015) is equivalent to the following conjecture: if V is a finite-dimensional subspace of polynomials such that all complex zeros of the Wronskian of V are real and negative, then all Plücker coordinates of V are positive. This conjecture is a totally positive strengthening of a result of Mukhin, Tarasov, and Varchenko (2009), and can be reformulated as saying that all complex solutions to a certain family of Schubert problems in the Grassmannian are real and totally positive.

HANNAH LARSON, Stanford University

Brill–Noether theory over the Hurwitz space

The Brill–Noether theorem describes the maps of general curves to projective space. In particular, when $g = (r+1)(g-d+r)$, a general genus g curve C admits a finite number of degree d maps $C \to \mathbb{P}^r$. The number of such maps has a nice combinatorial interpretation. I will discuss an analogue of this result, but for curves C already equipped with a map $C \to \mathbb{P}^1$ (the presence of such a map might force C to be special and so fail the Brill–Noether theorem). In joint work with E. Larson and I. Vogt, we answer an analogous enumeration problem by relating it to the combinatorics of the affine symmetric group.

SHIYUE LI, Brown University

Simple connectivity and intersection theory of moduli spaces of tropical weighted stable curves

Tropical moduli spaces of weighted stable curves are moduli spaces of metric weighted marked graphs satisfying certain stability conditions. I will present an inductive proof of the simple connectivity of these moduli spaces of curves of higher genus, which demonstrates the recursive structure of these symmetric Δ-complexes. I will then share with you a combinatorial result in tropical intersection theory on these moduli spaces; that is, a product decomposition formula of the weight of a maximal cone in an arbitrary-dimensional intersection of psi-classes into tropical Gromov-Witten multiplicities. This computation confirms the expectation that the classical and the tropical intersection numbers coincide in top-dimensional intersections, and provides a combinatorial perspective for arbitrary-dimensional intersections of psi-classes.

OLIVER PECHENIK, University of Waterloo

A web basis of invariant polynomials from noncrossing partitions

The irreducible representations of the symmetric group are called Specht modules S^λ and are indexed by partitions. We can realize S^λ as a certain graded piece of a ring of invariants, equivalently as global sections of a line bundle on a partial flag variety. There are many general ways to choose useful bases of this module. Particularly powerful are web bases, which make connections with cluster algebras and quantum link invariants, except that web bases are only available in very special cases; essentially, we only know web bases in the cases $\lambda = (m, m)$ and $\lambda = (m, m, m)$. Building on work of B. Rhoades, we find what appears to be a web basis of invariants for a special family of Specht modules with lambda of the form $(a, a, 1^b)$. The planar diagrams that appear are noncrossing set partitions, and we thereby obtain geometric interpretations of earlier enumerative results in tableau dynamics. (Joint work with Becky Patrias and Jessica Striker.)

COLLEEN ROBICHAUX, University of Illinois at Urbana-Champaign

Castelnuovo-Mumford regularity of ladder determinantal ideals via Grothendieck polynomials

We give degree formulas for Grothendieck polynomials indexed by vexillary permutations. We apply our formulas to compute the Castelnuovo–Mumford regularity of classes of generalized determinantal ideals. In particular, we give combinatorial formulas for the regularities of all one-sided mixed ladder determinantal ideals.

We also derive formulas for the regularities of certain Kazhdan–Lusztig ideals, including those coming from open patches of Grassmannians. This provides a correction to a conjecture of Kummini–Lakshmibai–Sastry–Seshadri (2015). This is joint work with Jenna Rajchgot and Anna Weigandt.

ROB SILVERSMITH, University of Warwick
Cross-ratios and perfect matchings

A collection $\mathcal{T} = \{T_1, T_2, \ldots, T_{n-3}\}$ of 4-element subsets of $[n]$ defines a product of forgetful maps $\mathcal{M}_{0,n} \to (\mathcal{M}_{0,4})^{n-3}$. The degree of this map is a nonnegative integer called the cross-ratio degree $d_{\mathcal{T}}$ of $\mathcal{T}$. It would be desirable to understand how $d_{\mathcal{T}}$ depends on the combinatorial structure of $\mathcal{T}$ as a hypergraph. I'll discuss several interpretations of cross-ratio degrees in algebra, algebraic geometry, and tropical geometry, and present a perhaps-surprising upper bound for cross-ratio degrees in terms of perfect matchings.