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The average size of 3-torsion in class groups of 2-extensions

We determine the average size of the 3-torsion in class groups of G -extensions of a number field when G is any transitive 2-group containing a transposition, for example D_4 . It follows from the Cohen–Lenstra–Martinet heuristics that the average size of the p -torsion in class groups of G -extensions of a number field is conjecturally finite for any G and most p (including $p \nmid |G|$). Previously this conjecture had only been proven in the cases of $G = S_2$ with $p = 3$ and $G = S_3$ with $p = 2$. We also show that the average 3-torsion in a certain relative class group for these G -extensions is as predicted by Cohen and Martinet, proving new cases of the Cohen–Lenstra–Martinet heuristics. Our new method also works for many other permutation groups G that are not 2-groups. (Joint with Jiuya Wang and Melanie Matchett Wood.)