JUNCHENG WEI, University of British Columbia
Stability of Sobolev Inequality

We consider the stability of Sobolev inequality at the critical point level. Suppose $u \in \dot{H}^1(\mathbb{R}^n)$. In a seminal work, Struwe proved that if $u \geq 0$ and $\Gamma(u) := \|\Delta u + u^{\frac{n+2}{n-2}}\|_{H^{-1}} \to 0$ then $dist(u, \mathcal{T}) \to 0$, where $dist(u, \mathcal{T})$ denotes the $\dot{H}^1(\mathbb{R}^n)$-distance of u from the manifold of sums of Talenti bubbles. Ciraolo, Figalli and Maggi obtained the first quantitative version of Struwe's decomposition with one bubble in all dimensions, namely $dist(u, \mathcal{T}) \leq C\Gamma(u)$. For Struwe's decomposition with two or more bubbles, Figalli and Glaudo showed a striking dimensional dependent quantitative estimate, namely $dist(u, \mathcal{T}) \leq C\Gamma(u)$ when $3 \leq n \leq 5$ while this is false for $n \geq 6$. In this talk, I will present our estimates in higher dimensions:

$$dist(u, \mathcal{T}) \leq C \begin{cases} \Gamma(u) \left|\log \Gamma(u)\right|^{\frac{1}{2}} & \textit{if } n = 6, \\ |\Gamma(u)|^{\frac{n+2}{2(n-2)}} & \textit{if } n \geq 7. \end{cases}$$

Furthermore, we show that this inequality is sharp. Extensions to Caffarelli-Kohn-Nirenberg inequalities, harmonic map inequality and half-harmonic map inequality will also be discussed.