Probability in Number Theory
Applications des Probabilités en Théorie des Nombres
(Org: **Louis-Pierre Arguin** and/et **Andrew Granville** (Université de Montréal))

EMMA BAILEY, University of Bristol
Random matrices and L-functions: moments of moments, branching, and log-correlation

Recently there has been a great deal of interest in understanding the moments of partition functions of logarithmically correlated processes. Particular examples of note are the logarithm of the zeta function over short intervals, the logarithm of characteristic polynomials of unitary matrices, and random walks on binary trees. In this talk I will present results for the moments of partition functions of such processes. This analysis also leads to conjectures for other families of primitive L-functions. This is joint work with Jon Keating and Theo Assiotis.

PAUL BOURGADE, Courant Institute, New York University
The Fyodorov-Hiary-Keating Conjecture

By analogy with conjectures for random matrices, Fyodorov-Hiary-Keating proposed precise asymptotics for the maximum of the Riemann zeta function in a typical short interval on the critical line. I will explain a recent proof of the upper bound part of their prediction. This is joint work with Louis-Pierre Arguin and Maksym Radziwill.

CLAIRE BURRIN, ETH Zurich
Higher moment formulas for discrete lattice orbits in the plane

We consider discrete sets in the plane arising from the linear action of a lattice in $\mathrm{SL}_2(\mathbb{R})$. The set of primitive integers vectors (i.e., where the coordinates are coprime) is one such example. In a very different direction, the set of holonomy vectors of saddle connections on a square-tiled surface provides another example. How are such discrete planar sets distributed in the plane ? I will report on on-going work with Samantha Fairchild.

FRANCESCO CELLAROSI, Queen's University
Rational Horocycle lifts and the tails of Quadratic Weyl sums

Equidistribution of horocycles on hyperbolic surfaces has been used to dynamically answer several probabilistic questions about number-theoretical objects. In this talk we focus on horocycle lifts, i.e. curves on higher-dimensional manifolds whose projection to the hyperbolic surface is a classical horocycle, and their behaviour under the action of the geodesic flow. It is known that when such horocycle lifts are 'generic', then their push forward via the geodesic flow becomes equidistributed in the ambient manifold. We consider certain 'non-generic' (i.e. rational) horocycle lifts, in which case the equidistribution takes place on a sub-manifold. We then use this fact to study the tail distribution of quadratic Weyl sums when one of their arguments is random and the other is rational. In this case we obtain random variables with heavy tails, all of which only possess moments of order less than 4. Depending on the rational argument, we establish the exact tail decay, which can be described with the help of the Dedekind ψ-function. Joint work with Tariq Osman.

JEAN-MARIE DE KONINCK, Université Laval
Consecutive integers divisible by a power of their largest prime factor

Given integers $k \geq 2$ and $\ell \geq 2$, the Chinese Remainder Theorem guarantees the existence of k consecutive integers divisible respectively by preassigned prime powers p_j^ℓ, $j = 1, \ldots, k$. However, there is no guarantee that the respective largest prime factors of the resulting k consecutive integers will be precisely the chosen p_j's. How can we make it so? Using elementary, analytic and probabilistic approaches, we shed some light and raise many questions regarding this difficult problem. This is joint work with Matthieu Moineau.

VESSELIN DIMITROV, University of Toronto
Small algebraic points and random polynomials: a distributional parallel and some recent results

I will draw a parallel between these two areas that features the distribution, interlacing and shadowing eccentricities among the zeros or the critical points of the respective polynomials. On either side of the analogy we consider some old and new results, with mutually motivated proofs, and hope to make a point that a useful interaction goes both ways. Besides the context of small Mahler measure, we also discuss the sample of Weil numbers and end with a hypothetical analog for the nearest matching of zeros within a family of L-functions.

SURESH ESWARATHASAN, Dalhousie University
Counting tangencies of nodal domains

Fix a smooth vector field V, with finitely many zeroes, on a compact surface $(\mathcal{M}, g)$ without boundary. We give results on the distribution of the number of tangencies to V of the nodal components of random band-limited functions. In the high-energy limit, the distributions obey a universal deterministic law, independent of the surface $\mathcal{M}$ and the vector field V. Applications towards arithmetic random waves on the flat torus will be discussed. This is joint work with I. Wigman (King's College London).

ADAM HARPER, University of Warwick
Large fluctuations of random multiplicative functions

Random multiplicative functions $f(n)$ are a well studied random model for deterministic multiplicative functions like Dirichlet characters or the Mobius function. Arguably the first question ever studied about them, by Wintner in 1944, was to obtain almost sure bounds for the largest fluctuations of their partial sums $\sum_{n \le x} f(n)$, seeking to emulate the classical Law of the Iterated Logarithm for independent random variables. It remains an open question to sharply determine the size of these fluctuations, and in this talk I will describe a new result in that direction.

WINSTON HEAP, Max Planck Institute for Mathematics
Random multiplicative functions and a model for the Riemann zeta function

We look at a weighted sum of random multiplicative functions and view this as a model for the Riemann zeta function. We investigate various aspects including its high moments, distribution and maxima.

DIMITRIS KOUKOULOPOULOS, Université de Montréal
How concentrated can the divisors of a typical integer be?

The Delta function measures the concentration of the sequence of divisors of an integer. Specifically, given an integer n, we write $\Delta(n)$ for the maximum over y of the number of divisors of n lying in the dyadic interval $[y, 2y]$. It was introduced by Hooley in 1979 because of its connections to various problems in Diophantine equations and approximation. In 1984, Maier and Tenenbaum proved that $\Delta(n) > 1$ for almost all integers n, thus settling a 1948 conjecture due to Erdős. In subsequent work, they proved that $(\log\log n)^{c+o(1)} \le \Delta(n) \le (\log\log n)^{\log 2+o(1)}$, where $c = (\log 2)/\log(\frac{1-1/\log 27}{1-1/\log 3}) \approx 0.33827$ for almost all integers n. In addition, they conjectured that $\Delta(n) = (\log\log n)^{c+o(1)}$ for almost all n. In this talk, I will present joint work with Kevin Ford and Ben Green that disproves the Maier-Tenenbaum conjecture by replacing the constant c in the lower bound by another constant $c' = 0.35332277\ldots$ that we believe is optimal. We also prove analogous results about permutations and polynomials over finite fields by reducing all three cases to an archetypal probabilistic model.

YOUNESS LAMZOURI, Université de Lorraine
Zeros of linear combinations of L-functions near the critical line

In this talk I will present a recent joint work with Yoonbok Lee, where we investigate the number of zeros of linear combinations of L-functions in the vicinity of the critical line. More precisely, we let $L_1, \dots, L_J$ be distinct primitive L-functions belonging to a large class (which conjecturally contains all L-functions arising from automorphic representations on $\mathrm{GL}(n)$) and $b_1, \dots, b_J$ be real numbers. Our main result is an asymptotic formula for the number of zeros of $F(s) = \sum_{j \le J} b_j L_j(s)$ in the region $\mathrm{Re}(s) \ge 1/2 + 1/G(T)$ and $\mathrm{Im}(s) \in [T, 2T]$, uniformly in the range $\log\log T \le G(T) \le (\log T)^{\nu}$, where $\nu \asymp 1/J$. This establishes a generalization of a conjecture of Hejhal in this range.

YU-RU LIU, University of Waterloo
Number of Prime Factors with a Given Multiplicity

In this talk, we study a variation of the ω function. More precisely, given the positive integer k, let $\omega_k(n)$ denote the number of distinct prime factors of n which occur with multiplicity k. We will prove that $\omega_1(n)$ has the normal order $\log\log n$, while $\omega_k(n)$ does not have normal order. This is joint work with Ertan Elma.

SACHA MANGEREL, Centre de Recherche Mathématiques
Arrangements of Consecutive Values of Real Multiplicative Functions

We will discuss the following problem: given a multiplicative function $f : \mathbb{N} \to \mathbb{R}$ and a k-tuple of "admissible", distinct non-negative integer shifts $a_1, \dots, a_k$, what is the probability that a given $n \in \mathbb{N}$ satisfies $f(n + a_1) \le \cdots \le f(n + a_k)$? Randomness heuristics suggest that such a pattern occur with probability $1/k!$ for a "generic" function f. Under certain assumptions on f we will give both conditional and unconditional results in this direction for a large collection of examples, in particular the Ramanujan τ function as well as sequences of Fourier coefficients of many non-CM, arithmetically normalized Hecke eigencusp forms with trivial nebentypus.

RAM MURTY, Queen's University
An "all-purpose" Erdos-Kac theorem

We will discuss a general axiomatic formulation that allows for the derivation of Erdos-Kac type theorems in a wide range of contexts. In particular, we will apply it to derive Erdos-Kac type theorems for the number of prime factors of sums of Fourier coefficients of Hecke eigenforms. This is joint work with Kumar Murty and Sudhir Pujahari.

MICHEL PAIN, Courant Institute (NYU)
Extrema of branching random walks and log-correlated fields

In this talk, I will discuss in an introductory manner the study of extrema of branching random walks and its application to the wider class of log-correlated fields, which includes the logarithm of the Riemann zeta function on short intervals of the critical line and the logarithm of the characteristic polynomial of random matrices. Moreover, I will present some recent results that fit into this general picture.

MAKSYM RADZIWILL, Caltech
Moments of the Riemann zeta function in tiny intervals

I will discuss work with Frederic Ouimet and Louis-Pierre Arguin on the behavior of moments of the Riemann zeta-function in typical (tiny) intervals.

BRAD RODGERS, Queen's University
The distribution of sums of two squares in short intervals

In this talk I will discuss the distribution in short intervals of integers representable as sums of two squares. For sufficiently short intervals this distribution is (conjecturally) governed by a Poisson distribution, but I will explain why one should expect in intervals which are just a little longer a connection to what are known as z-measures. These were first investigated in the context of harmonic analysis on the infinite symmetric group and I hope to also give a short introduction to them. Results can be proved in a function field setting. This is joint work with Ofir Gorodetsky.

CAMERON STEWART, University of Waterloo
Counting solvable S-unit equations

We shall discuss joint work with I. Shparlinski concerning upper bounds on the number of finite sets S of primes below a given bound for which various 2 variable S-unit equations have a solution.

ALED WALKER, Centre de Recherches Mathématiques
Triple correlations of dilates squares modulo 1

Twenty years ago, Rudnick-Sarnak-Zaharescu made a deep conjecture about the gap distribution of αn^2 modulo 1. They posited that this distribution should be poissonian, provided some generic conditions on the diophantine approximation of the dilate α were satisfied. In this talk I will give a summary of the previous work around this conjecture, and describe a recent result which extends the threshold for the triple correlations beyond the trivial range. This is joint work with Niclas Technau.

ASIF ZAMAN, University of Toronto
Low moments of random power series

Harper recently proved that the low moments of partial sums of random multiplicative functions exhibit better-than-squareroot cancellation, as conjectured by Helson. This breakthrough result was surprising and its proof is a serious technical feat. Together with Soundararajan, we establish a closely related result for low moments of certain random power series. Our proof possesses the same key principles and phenomena as Harper's but our idealized setting affords several simplifications to the arguments. I will discuss the setup and share some of the key simplifications.