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Density hypothesis in horizontal families

Let G be a real semi simple Lie group with an irreducible unitary representation π. The non-temperedness of π is measured by the real parameter $p(\pi)$ which is defined as the infimum of $p \geq 2$ such that π has non-zero matrix coefficients in $L^p(G)$. Sarnak and Xue conjectured that for any arithmetic lattice $\Gamma \subset G$ and principal congruence subgroup $\Gamma(q) \subset \Gamma$, the multiplicity of π in $L^2(G/\Gamma(q))$ is at most $O(V(q)^{2/p(\pi)+\epsilon})$ where $V(q)$ is the covolume of $\Gamma(q)$. Sarnak and Xue proved this conjecture for $G = SL(2,R), SL(2,\mathbb{C})$. In a a joint work with Gergely Harcos, Peter Maga and Djordje Milicevic we prove bounds of the same quality that hold uniformly for families of pairwise non-commensurable lattices in $G = SL(2,\mathbb{R})^a \times SL(2,\mathbb{C})^b$ given as unit groups of maximal orders of quaternion algebras over number fields ("horizontal families"). I will also discuss how the multiplicity bounds depend on the Archimedean parameters and some possible extensions of our methods.