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The Central Value of Dirichlet L-functions over Rational Function Fields

The central value of a Dirichlet L-function over $\mathbb{F}_q(t)$ is governed by the Zeta function of a smooth project curve over $\mathbb{F}_q$. Using this connection to geometry, we show a lower bound on the number of quadratic characters with conductor $\leq X$ whose L-functions vanish at the central point. The existence of infinitely many such characters is in contrast with the situation over the rational numbers, where a conjecture of Chowla predicts there should be no such L-functions. Towards this direction, for each fixed q, we give an explicit upper bound on the number of such quadratic characters. This upper bound decreases as q grows and it goes to 0% as $q \to \infty$. In this talk, I will also discuss Dirichlet characters of odd prime order ℓ and the central value of their L-functions. Some of the results in this talk are joint work with Jordan Ellenberg and Mark Shusterman.