ARUL SHANKAR, University of
The 2-torsion subgroups of the class groups in families of cubic fields

The Cohen–Lenstra–Martinet conjectures have been verified in only two cases. Davenport–Heilbronn compute the average size of the 3-torsion subgroups in the class group of quadratic fields and Bhargava computes the average size of the 2-torsion subgroups in the class groups of cubic fields. The values computed in the above two results are remarkably stable. In particular, work of Bhargava–Varma shows that they do not change if one instead averages over the family of quadratic or cubic fields satisfying any finite set of splitting conditions.

However for certain "thin" families of cubic fields, namely, families of monogenic and n-monogenic cubic fields, the story is very different. In this talk, we will determine the average size of the 2-torsion subgroups of the class groups of fields in these thin families. Surprisingly, these values differ from the Cohen–Lenstra–Martinet predictions! We will also provide an explanation for this difference in terms of the Tamagawa numbers of naturally arising reductive groups. This is joint work with Manjul Bhargava and Jon Hanke.