Symplectic Topology
Topologie Symplectique
(Org: **Octav Cornea** and/et **Egor Shelukhin** (Université de Montréal))

MARCELO ATALLAH, Université de Montréal
Hamiltonian no-torsion

Abstract: In 2002 Polterovich notably showed that Hamiltonian diffeomorphisms of finite order, which we call Hamiltonian torsion, must be trivial on closed symplectically aspherical manifolds. We study the existence of Hamiltonian torsion and its metric rigidity properties in more general situations. First, we extend Polterovich's result to closed symplectically Calabi-Yau and closed negative monotone manifolds. Second, going beyond topological constraints, we describe how Hamiltonian torsion is related to the existence of pseudo-holomorphic spheres and answer a close variant of Problem 24 from the introductory monograph of McDuff-Salamon. Finally, we prove an analogue of Newman's 1931 theorem for Hofer's metric and Viterbo's spectral metric on the Hamiltonian group of monotone symplecitc manifolds: a sufficiently small ball around the identity contains no torsion. During the talk, I shall discuss the results above and some of the key ingredients of their proofs. This talk is based on joint work with Egor Shelukhin.

PRANAV CHAKRAVARTHY, University of Western Ontario
Homotopy type of equivariant symplectomorphisms of rational ruled surfaces.

Darboux's theorem states that "all symplectic manifolds locally look alike". Consequently, there are no local invariants in symplectic geometry, and one must look for global invariants to probe symplectic manifolds. Such invariants can be obtained by investigating the homotopy type of mapping spaces related to the symplectic structure. In this talk, we compute the homotopy type of the group of equivariant symplectomorphisms of $S^2 \times S^2$ and $\mathbb{C}P^2$ blown up once under the presence of hamiltonian S^1 or finite cyclic group actions.

JEAN-PHILIPPE CHASSÉ, Université de Montréal
The impact of metric constraints on the behavior of shadow metrics

Since its introduction by Hofer, the eponymous norm on the group of Hamiltonian diffeomorphisms has been of great importance in symplectic topology. In particular, the Hofer norm induces a metric on the Hamiltonian orbit of a Lagrangian submanifold, as proved by Chekanov. However, this gives no way to significantly compare Lagrangian submanifolds which are not Hamiltonian diffeomorphic, let alone those which do not have the same homotopy type. This is one of the reasons why the so-called shadow metrics — or more generally the weighted fragmentation pseudometrics — introduced by Biran, Cornea and Shelukhin, are a very interesting and promising object of study.

After a brief explainer on shadow metrics, I will present a conjecture of Cornea on how they are related to the set-theoretic Hausdorff distance when one looks at a subspace of Lagrangian submanifolds respecting certain metric constraints. I will then present a proof of the conjecture in certain cases based on Groman and Solomon's reverse isoperimetric inequality. If time permits, I will explain how this result extends to other weighted fragmentation pseudometrics.

FRANCISCO TORRES DE LIZAUR, University of Toronto
Knots and links in Beltrami fields

Beltrami fields on a 3-dimensional compact manifold are eigenfields of the curl operator. They describe a stationary ideal fluid whose vorticity and velocity are aligned. In this talk I will show that, on the round 3-sphere and the flat 3-torus, there are Beltrami fields having a finite set of periodic orbits and invariant tori of any given knot and link type, provided the eigenvalue is large enough. This is joint work with Alberto Enciso and Daniel Peralta-Salas.

ZHANG JUN, University of Montreal
Quantitative Lagrangian embeddings

Symplectic embedding between domains is a central problem in symplectic geometry. In this talk, we will discuss a different type of embedding - Lagrangian embedding, as well as its resulting obstructions to symplectic embeddings of basic domains (for instance, symplectic embeddings from 4-dimensional polydiscs to ellipsoids). The key tool is the shape invariant, a collection of quantitative data (called area classes) of Lagrangian embeddings. The main theorem in this talk is a computational result of the shape invariant of a large family of 4-dimensional ellipsoids. The computation is based on the symplectic field theory (SFT) and embedded contact homology (ECH) theory. This talk is based on joint work with Richard Hind.

ILIA KIRILLOV, University of Toronto
Classification of coadjoint orbits for symplectomorphism groups of surfaces with boundary

Hydrodynamical Euler's equation describes the motion of an ideal incompressible fluid on a Riemannian manifold. In this talk, I will start by explaining how the kinematics of Euler's equation is related to the coadjoint orbits of the group of volume-preserving diffeomorphisms. In dimension two the volume-preserving diffeomorphisms coincide with the symplectomorphisms. The classification of generic coadjoint orbits for symplectomorphism groups of closed surfaces was obtained by Izosimov, Khesin, and Mousavi in 2016. I will explain how to generalize this result to the case of symplectic surfaces with boundary.

LARA SUAREZ LOPEZ, Ruhr Universitaet Bochum
On the rigidity of Legendrian cobordisms

It is a natural question in differential topology whether an isotopy invariant of a manifold is also a cobordism invariant. In symplectic and contact topology there are different notions of cobordisms. One of them due to Arnold concerns Lagrangian and Legendrian cobordisms between Lagrangian/Legendrian submanifolds. For Lagrangian cobordisms, Biran-Cornea showed that monotone ones preserve Floer homology. In the first part of this talk I will show a similar statement for Legendrian cobordisms that are hypertight. Then I will talk about positive Legendrian cobordism. This last part is based on a joint project with Maÿlis Limouzineau.

JORDAN PAYETTE, Université de Montréal and CIRGET
Mean value inequalities for the Poisson bracket invariant

In this talk, motivated by a conjecture of Polterovich from 2012, I shall discuss the existence of the following mean value phenomenon in symplectic topology: "The more localized the supports of the functions forming a partition of unity on a symplectic manifold are, the more Poisson-noncommutative the functions are."

I shall first present an elementary approach to prove "mean value inequalities" for closed symplectic surfaces. For surfaces of genus at least one, this implies Polterovich's conjecture – which has been recently established for all closed surfaces by Buhovsky–Logunov–Tanny by other means. I shall then describe a work in progress with L. Buhovsky and S. Tanny aiming to use pseudoholomorphic curves to deduce "mean value inequalities" for higher-dimensional symplectic manifolds from the known two-dimensional results.

DOMINIQUE RATHEL-FOURNIER, Université de Montréal
Unobstructed Lagrangian cobordism groups of surfaces

The Lagrangian cobordism groups of a symplectic manifold encode the relations between Lagrangian submanifolds given by suitable classes of Lagrangian cobordisms. Biran and Cornea showed that, in certain circumstances, there is a natural morphism from the Lagrangian cobordism group to the Grothendieck group of the derived Fukaya category of the manifold.

In this talk, we consider the case of a surface of genus $g \geq 2$. In the first part, we show how to extend the result of Biran and Cornea to a class of immersed cobordisms satisfying a non-obstruction assumption.

In the second part, we show that in this case the morphism from the cobordism group to $K_0(\mathrm{DFuk})$ is an isomorphism. The proof builds upon previous work of Perrier, and also relies on Abouzaid's computation of $K_0(\mathrm{DFuk})$ for higher genus surfaces. Our main contribution is the proof that a large class of surgery cobordisms are topologically unobstructed, in the sense that they do not bound non-trivial disks or teardrops.

XIUDI TANG, University of Toronto
Symplectic ray removal

We remove a properly embedded ray from a noncompact symplectic manifold without changing the symplectic structure.

Reference: https://arxiv.org/abs/1812.00444.

SHIRA TANNY, Tel Aviv University
The Poisson bracket invariant: elementary and hard approaches.

In 2006 Entov and Polterovich proved that functions forming a partition of unity with displaceable supports cannot commute with respect to the Poisson bracket. In 2012 Polterovich conjectured a quantitative version of this theorem. I will discuss three interconnected topics: a solution of this conjecture in dimension two (with Lev Buhovsky and Alexander Logunov), a link between this problem and Grothendieck's theorem from functional analysis (with Efim Gluskin), and new results related to the Floer-theoretical approach to this conjecture (with Yaniv Ganor).

QUN WANG, Toronto
Choreographies in the N-Vortex Problem

Initially emerged from the study of the N-body problem in celestial mechanics, the choreographies are periodic orbits in which all the bodies are equally spread out along a single trajectory. In this talk, we study the existence of the choreographies in the N-vortex problem arising from the Euler equation. The identical vorticity permits us to construct the symmetric holomorphic spheres. Using this tool we prove that there exist infinitely many non-trivial choreographies for the identical n-vortex problem.

CHENG YANG, University of Toronto
Symplectic reduction and perturbation theory

In this talk, we show that the averaged equation for a one-frequency fast-oscillating Hamiltonian system is the result of symplectic reduction of a certain natural system on the corresponding S^1-bundle with respect to the circle action. Furthermore, if the reduced configuration space happens to be a group, then under natural assumptions the averaged system turns out to be the Euler equation on a central extension of that group. This gives a new explanation of the drift, common in averaged system, as a similar shift is typically present in symplectic reductions and central extensions. This is a joint work with Boris Khesin.