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The ultraspherical spectral element method

We introduce a novel spectral element method based on the ultraspherical spectral method and the hierarchical Poincaré–Steklov scheme for solving second-order linear partial differential equations on polygonal domains with unstructured quadrilateral or triangular meshes. Properties of the ultraspherical spectral method lead to almost banded linear systems, allowing the element method to be competitive in the high-polynomial regime ($p > 5$). The hierarchical Poincaré–Steklov scheme enables precomputed solution operators to be reused, allowing for fast elliptic solves in implicit and semi-implicit time-steppers. The resulting spectral element method achieves an overall computational complexity of $\mathcal{O}(p^4/h^3)$ for mesh size h and polynomial order p, enabling hp-adaptivity to be efficiently performed. We develop an open-source software system, ultraSEM, for flexible, user-friendly spectral element computations in MATLAB. Joint work with Alex Townsend (Cornell University) and Nicholas Hale (Stellenbosch University).