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Twice is enough for dangerous eigenvalues

A popular class of methods for large-scale eigenvalue problems use Cauchy's integral formulas to compute eigenvalues of a large matrix in a target region. We analyze the stability of these methods in the singular limit, i.e., as eigenvalues of the matrix approach the contour. Remarkably, contour-integral eigensolvers that incorporate subspace iterations are stable: the "dangerous eigenvalues" near the contour contribute large round-off errors in the first iteration, but are self-correcting in later iterations. For matrices with orthogonal eigenvectors (e.g., real-symmetric or complex Hermitian), two iterations is enough to reduce round-off errors to the order of the unit-round off. In contrast, contour-integral eigensolvers that construct Krylov subspaces typically fail to converge to unit round-off accuracy when an eigenvalue is close to the contour. However, we suggest a simple new restart strategy that recovers full precision in the target eigenpairs after two iterations.