Geometric and Computational Spectral Theory
Théorie Spectrale Géométrique et Computationnelle
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BENJAMIN BOGOSEL, Centre de Mathématiques Appliquées, Ecole Polytechnique
Shape optimization of the Steklov eigenvalues under various constraints

Many recent works deal with the shape optimization of the Steklov eigenvalues. There are only a few cases where the optimal shapes are explicitly known, which motivates the study of numerical algorithms that can approximate efficiently the optimal shapes. In this presentation theoretical aspects regarding the existence of optimal shapes and numerical aspects regarding the numerical computation of Steklov eigenvalues are shown. Algorithms for optimizing numerically functionals related to the Steklov eigenvalues under volume, diameter and convexity constraints are also presented.

This work is the result of collaborations with D. Bucur, A. Giacomini, A. Henrot, F. Nacry and A. Al Sayed.

OSCAR BRUNO, Caltech
Domains Without Dense Steklov Nodal Sets

This talk concerns the asymptotic geometric character of the nodal set of the eigenfunctions of the Steklov eigenvalue problem in two-dimensional domains. In particular results will be mentioned which establish the existence of a dense family $\mathcal{A}$ of simply-connected two-dimensional domains with analytic boundaries for each one of which the Steklov eigenfunction's nodal lines "are not dense at scale $1/j$". This result, which addresses a question put forth under "Open Problem 10" in Girouard and Polterovich, J. Spectr. Theory, 321-359 (2017), shows that, for domains in the class $\mathcal{A}$, the Steklov nodal sets have starkly different character than anticipated: they are not dense at any shrinking scale. A variety of numerical results, including surprising graphical manifestations of the non-dense nodal character, will also be presented. Work in collaboration with Jeffrey Galkowski.

GRAHAM COX, Memorial University
Defining the spectral position of a Neumann domain

A Laplacian eigenfunction on a Riemannian surface provides a natural partition into Neumann domains—open subsets on which the function satisfies Neumann boundary conditions. These are a natural analogue of nodal domains, on which the eigenfunction satisfies Dirichlet boundary conditions, but their analysis ends up being much more involved.

In this talk I will explain why, on a given Neumann domain, the Neumann Laplacian has compact resolvent (and hence discrete spectrum), and the restricted eigenfunction is an eigenfunction of the Neumann Laplacian. The difficulty in proving these results is that the boundary of a Neumann domain may have cusps and cracks, and hence is not necessarily continuous, so standard density and compactness results for Sobolev spaces are not available. This problem can be overcome using special geometric properties of the Neumann domain boundary, which is made up of gradient flow lines for the corresponding eigenfunction.

These results allow one to define the spectral position of a Neumann domain. (Unlike a nodal domain, the restricted eigenfunction on a Neumann domain is never the ground state.) Finally, I will present a formula for computing the spectral position using the Dirichlet-to-Neumann map. This is joint work with Ram Band and Sebastian Egger.

SEBASTIAN DOMINGUEZ, University of Saskatchewan
Steklov eigenvalues in linear elasticity

In this talk we discuss Steklov eigenvalues for the Lamé operator in linear elasticity. In this eigenproblem the spectral parameter appears in a Robin-type boundary condition, linking the traction and the displacement. To establish the existence of a countable spectrum for this problem, we present an extension of Korn's inequality. We also show that a proposed conforming Galerkin scheme provides convergent approximations to the true eigenvalues. Finally, a standard finite element method is used to conduct numerical experiments on 2D and 3D domains.

EMILY DRYDEN, Bucknell University
Heat content of polygons

Imagine heating up a planar domain to uniform temperature and freezing its boundary at temperature zero. If we measure the heat content of the domain over time, what geometric information can we obtain? Does this geometric information uniquely determine planar domains? We will investigate these questions for planar polygons. This talk is based on joint work with Madelyne Brown and Jeffrey Langford.

SURESH ESWARATHASAN, Dalhousie University
Entropy of ϵ-logarithmic quasimodes

Consider (M, g) a hyperbolic surface without boundary and its semiclassical Laplacian $-\hbar^2\Delta_g$. It has been shown that for sequences of $-\hbar^2\Delta_g$-eigenfunctions $\{\psi_\hbar\}_\hbar$ (with central energy $E > 0$) and corresponding semiclassical measure μ_{sc}, the Kolmogorov-Sinai entropy $H_{KS}(\mu_{sc})$ is bounded below by $\frac{1}{2}$.
In this talk, we discuss the semiclassical measures μ_{sc} of special sums of $-\hbar^2\Delta_g$ eigenfunctions, namely *ϵ-logarithmic quasimodes* $\Psi_\hbar$ (with central energy $E > 0$) where $\epsilon > 0$. We show that for any $c \in [0, \frac{1}{2}]$, there exists $\epsilon = \epsilon(c)$ and a family of $\{\Psi_\hbar\}_\hbar$ of ϵ-logarithmic quasimodes whose $H_{KS}(\mu_{sc})$ is bounded below by c. This continues/generalizes some work of Anantharaman-Koch-Nonnenmacher, amongst others.

JEFFREY GALKOWSKI, University College London
Geodesic beams and Weyl remainders

In this talk we discuss quantitative improvements for Weyl remainders under dynamical assumptions on the geodesic flow. We consider a variety of Weyl type remainders including asymptotics for the eigenvalue counting function as well as for the on and off diagonal spectral projector. These improvements are obtained by combining the geodesic beam approach to understanding eigenfunction concentration together with an appropriate decomposition of the spectral projector into quasimodes for the Laplacian. One striking consequence of these estimates is a quantitatively improved Weyl remainder on all product manifolds. This is joint work with Y. Canzani.

ALEXANDRE GIROUARD, Université Laval
Planar domains with prescribed perimeter and large Steklov spectral gap must collapse to a point

In 2014, Gerasim Kokarev proved that the first nonzero Steklov eigenvalue of a compact surface Ω of genus 0 satisfies $\overline{\sigma}_1(\Omega) := \sigma_1(\Omega)|\partial\Omega| \leq 8\pi$. In a recent joint work with Jean Lagacé, we proved that this inequality is sharp by constructing a sequence of domains in the sphere $\mathbb{S}^2 \subset \mathbb{R}^3$ that saturates it. In an ongoing project with Mikhail Karpukhin and Jean Lagacé, we went further and proved the saturation of Kokarev inequality for planar domains: there exists a sequence $\Omega^\epsilon \subset \mathbb{R}^2$ such that $\overline{\sigma}_1(\Omega^\epsilon) \xrightarrow{\epsilon\to 0} 8\pi$. In this talk I will present a quantitative improvement of Kokarev's inequality, which sheds light on geometric and topological properties of such maximizing sequences for $\overline{\sigma}_1$. A particularly striking consequence is that any sequence $\Omega^\epsilon \subset \mathbb{R}^2$ with prescribed perimeter $|\Omega^\epsilon| = 1$ and $\sigma_1(\Omega^\epsilon) \xrightarrow{\epsilon\to 0} 8\pi$ accumulates at a point: Diameter$(\Omega^\epsilon) \xrightarrow{\epsilon\to 0} 0$. Another consequence is a quantitative lower bound on the number of connected components of the boundary $\partial\Omega$, which must grow to $+\infty$.

KATIE GITTINS, Durham University
Comparing Hodge spectra of manifolds and orbifolds: Part 2.

We consider the Hodge Laplacian acting on differential forms on closed Riemannian orbifolds. It is interesting to investigate whether it is possible to glean information about the singularities from the spectral data. We focus on whether orbifolds with singularities are spectrally distinguished from smooth manifolds. We apply the heat invariants for differential forms to obtain several positive results in this direction. For example, we obtain that the spectra of the Laplacian for functions and 1-forms together can detect the presence of singularities for orbifolds of dimension at most 3. Time-permitting, we may also discuss some negative results by presenting counterexamples.

This is based on joint work with Carolyn Gordon, Magda Khalile, Ingrid Membrillo Solis, Mary Sandoval and Elizabeth Stanhope.

CAROLYN GORDON,
Comparing Hodge spectra of manifolds and orbifolds: Part 1

We address the question: To what extent does spectral data encode information about the singularities of a closed Riemannian orbifold? Following a brief introduction to orbifolds and a partial history of results on the question above, we will focus on the Hodge Laplacian acting on differential forms. We address the small time heat asymptotics, paying particular attention to the contributions of the orbifold singularities to the heat invariants.

This talk is based on joint work with Katie Gittins, Magda Khalile, Ingrid Membrillo Solis, Mary Sandoval and Elizabeth Stanhope.

ASMA HASSANEZHAD, University of Bristol
Eigenvalue and multiplicity bounds for the mixed Steklov problem

We will discuss how some of the eigenvalue and multiplicity bounds known for the Steklov problem can be extended and improved for the mixed Steklov problem. The main focus will be in the 2-dimensional setting. But we also discuss the higher-dimensional setting when we consider eigenvalue bounds.

The main part of the talk is based on joint work with T. Arias-Marco, E. Dryden, C. Gordon, A. Ray and E. Stanhope.

DAVE HEWETT, University College London
Acoustic scattering by fractal screens

We study time-harmonic acoustic scattering in $\mathbb{R}^n$ $(n = 2, 3)$ by a fractal planar screen, assumed to be a non-empty bounded subset Γ of the hyperplane $\Gamma_\infty = \mathbb{R}^{n-1} \times \{0\}$. We consider two distinct cases: (i) Γ is a relatively open subset of Γ_∞ with fractal boundary (e.g. the interior of the Koch snowflake in the case $n = 3$); (ii) Γ is a compact fractal subset of Γ_∞ with empty interior (e.g. the Sierpinski triangle in the case $n = 3$). In both cases our numerical simulation strategy involves approximating the fractal screen Γ by a sequence of smoother "prefractal" screens, for which we compute the scattered field using boundary element methods that discretise the associated boundary integral equations. We prove sufficient conditions on the mesh sizes guaranteeing convergence to the limiting fractal solution, using the framework of Mosco convergence. We also provide numerical examples illustrating our theoretical results.

DIMA JAKOBSON, McGill
Zero and negative eigenvalues of conformally covariant operators, and nodal sets in conformal geometry

We first describe conformal invariants that arise from nodal sets and negative eigenvalues of conformally covariant operators (such as Yamabe or Paneitz operator). We discuss applications to curvature prescription problems. We prove that the Yamabe operator can have an arbitrarily large number of negative eigenvalues on any manifold of dimension greater than 2. We show that 0 is generically not an eigenvalue of the conformal Laplacian. If time permits, we shall discuss related results on manifolds with boundary, as well as for weighted graphs. This is joint work with Y. Canzani, R. Gover, R. Ponge, A. Hassannezhad, M. Levitin, M. Karpukhin, G. Cox and Y. Sire.

CHIU-YEN KAO, Claremont McKenna College
Computation of free boundary minimal surfaces via extremal Steklov eigenvalue problems

Recently Fraser and Schoen showed that the solution of a certain extremal Steklov eigenvalue problem on a compact surface with boundary can be used to generate a free boundary minimal surface, i.e., a surface contained in the ball that has (i) zero mean curvature and (ii) meets the boundary of the ball orthogonally (doi:10.1007/s00222-015-0604-x). We develop numerical methods that use this connection to realize free boundary minimal surfaces. Namely, on a compact surface, Σ, with genus γ and b boundary components, we maximize $\sigma_j(\Sigma, g)\ L(\partial\Sigma, g)$ over a class of smooth metrics, g, where $\sigma_j(\Sigma, g)$ is the j-th nonzero Steklov eigenvalue and $L(\partial\Sigma, g)$ is the length of $\partial\Sigma$. Our numerical method involves (i) using conformal uniformization of multiply connected domains to avoid explicit parameterization for the class of metrics, (ii) accurately solving a boundary-weighted Steklov eigenvalue problem in multi-connected domains, and (iii) developing gradient-based optimization methods for this non-smooth eigenvalue optimization problem. For genus $\gamma = 0$ and $b = 2, \ldots, 9, 12, 15, 20$ boundary components, we numerically solve the extremal Steklov problem for the first eigenvalue. The corresponding eigenfunctions generate a free boundary minimal surface, which we display in striking images. For higher eigenvalues, numerical evidence suggests that the maximizers are degenerate, but we compute local maximizers for the second and third eigenvalues with $b = 2$ boundary components and for the third and fifth eigenvalues with $b = 3$ boundary components. This is joint work with Braxton Osting (University of Utah) and Édouard Oudet (Université Grenoble Alpes).

MIKHAIL KARPUKHIN, California Institute of Technology
Continuity of eigenvalues with applications to eigenvalue optimization

In this talk we discuss a general framework of eigenvalues associated to Radon measures on Riemannian manifolds. This setup unifies a variety of classical eigenvalue problems, including Laplacian and Steklov problems. A simple condition on a sequence of measures μ_n that guarantees the continuity of corresponding eigenvalues is provided. We give applications to eigenvalue optimization problems. The talk is based on a joint work with J. Lagacé and A. Girouard.

JEAN LAGACÉ, University College London
Geometric homogenisation theory and spectral shape optimisation

In this talk, I will discuss how we can obtain upper bounds for Laplace eigenvalues from bounds for Steklov eigenvalues. This will be done through geometric homogenisation methods, in order to approximate the Laplace spectrum of any surface by the Steklov spectrum of a domain in that surface. The usual theory of homogenisation uses the periodic structure of Euclidean space to describe limits of singular problems, and I will discuss how it can be adapted deterministically to a setting without notions of periodicity.

HENRIK MATHIESEN, University of Chicago
Free boundary minimal surfaces of any topological type in Euclidean balls via shape optimization (Part 2)

This is a continuation of R. Petrides' talk giving some more applications of our recent work on gap results in glueing constructions for eigenvalues.

I will discuss two further consequences. How our glueing results give some information on the asymptotic behaviour of the free boundary minimal surfaces associated to maximizers for the first Steklov eigenvalue, and our proof of rigidity of the first conformal Steklov eigenvalue on annuli.

ANTOINE METRAS, University of Montreal
Steklov extremal metrics in higher dimension

Since the original papers of Fraser and Schoen in 2012, which highlighted the relation between extremal metrics for the Steklov normalized eigenvalue $\bar{\sigma}(\Sigma, g) = \sigma(\Sigma, g)|\partial\Sigma|$ on surface and free boundary minimal surface in B^m, those two subjects have been highly studied.

In higher dimension $n \geq 3$, how the Steklov eigenvalues should be normalized (by volume? boundary volume? a mixed of both?) is not a priori clear. In this talk I will discuss how only one choice of normalization allows for Stekov extremal metrics. I will also talk about the connection between Steklov conformal-extremal metrics on M^n, n-harmonic maps and the need to consider the Steklov problem with boundary density.

Based on joint work with Mikhail Karpukhin.

BRAXTON OSTING, University of Utah
Maximal Spectral Gaps for Periodic Schroedinger Operators

The spectrum of a Schroedinger operator with periodic potential generally consists of bands and gaps. In this paper, for fixed m, we consider the problem of maximizing the gap-to-midgap ratio for the m-th spectral gap over the class of potentials which are pointwise bounded and have fixed periodicity. In one dimension, we prove that the optimal potential is a unique step-function attaining the imposed minimum and maximum values on exactly m intervals. In two-dimensions, we develop an efficient rearrangement method for this problem and apply it to study properties of extremal potentials. Using an explicit parametrization of two-dimensional Bravais lattices, we also consider how the optimal value varies over equal-area lattices. This is joint work with Chiu-Yen Kao.

JEFFREY OVALL, Portland State University
Exploring Eigenvector Localization Using Filtered Subspace Iteration (FEAST)

Domain geometry and properties of the coefficients of selfadjoint elliptic operators can cause certain eigenvectors to be highly localized in relatively small subdomains. Such localization phenomena have generated a lot of interest in the physics and mathematics communities since the late 1950s, but the underlying mechanisms driving localization are still not fully understood, despite advances on the mathematical side during the last decade. Computational approaches for identifying likely regions of localizations and approximating localized eigenvectors and/or their corresponding eigenvalues have emerged in the past two years. We present a new approach in which filtered subspace iteration is applied to a perturbed version of the selfadjoint operator, where the complex perturbation is chosen to highlight eigenvectors that are localized in a user-specified region. Preliminary theoretical and computational results will be presented.

ROMAIN PETRIDES, Université de Paris
Free boundary minimal surfaces of any topological type in euclidean balls via shape optimization (Part 1)

Maximal metrics for the isoperimetric problem for Steklov eigenvalues on Riemannian surfaces arise as induced metrics of free boundary minimal surfaces in Euclidean balls. Then it is natural to perform a variational method on Steklov eigenvalues in order to build new minimal surfaces. The program of building free boundary minimal surfaces into Euclidean balls of any topological type is now completed by this method. It is a consequence of new gap results on first eigenvalues with respect to the topology in a recent joint work with H. Matthiesen. I will give some idea of the glueing construction and the asymptotic analysis behind these results.

IOSIF POLTEROVICH, Université de Montréal
The Dirichlet-to-Neumann map, the boundary Laplacian and an unpublished paper of Hörmander

In late 1950s, Lars Hörmander wrote a paper on the connection between the Dirichlet-to-Neumann map and the boundary Laplacian. The manuscript has not been published until two years ago. Interestingly enough, it contains the main ideas needed to deduce the inequalities between the Steklov and Laplace eigenvalues, obtained in a series of recent articles starting with the work of Provenzano-Stubbe. We discuss the Hörmander's approach as well as some related results, in particular, on the asymptotics of Steklov eigenvalues for non-smooth domains. The talk is based on a joint work in progress with A. Girouard, M. Karpukhin and M. Levitin.

MARIO SCHULZ, Queen Mary University of London
Free boundary minimal surfaces in the unit ball

The study of extremals for Steklov eigenvalues has revitalised the theory of free boundary minimal surfaces. One of the most basic open questions can be phrased as follows: Can a surface of any given topology be realised as a free boundary minimal surface in the Euclidean unit ball? We will answer this question for surfaces with connected boundary and arbitrary genus.

DAVID SHER, DePaul University
Inverse Steklov spectral problem for curvilinear polygons

Consider the Steklov spectral problem on curvilinear polygons in $\mathbb{R}^2$. Assuming all angles are less than π, the high-energy asymptotics of the Steklov spectrum are known. Specifically, the spectrum is asymptotic to the zero set of an explicit trigonometric polynomial constructed from the side lengths and the angles of the polygon. Here we consider the corresponding inverse spectral problem. We show that the Steklov spectrum of a curvilinear polygon determines the number of vertices, the ordered sequence of side lengths, and - up to a natural equivalence relation - the angles of that curvilinear polygon. This is joint work with S. Krymski, M. Levitin, L. Parnovski, and I. Polterovich.

DANIEL STERN, University of Chicago
Shape optimization in spectral geometry via variational methods for harmonic maps

I'll describe joint work with Mikhail Karpukhin, relating the problem of maximizing Laplacian eigenvalues over unit-area metrics on a given Riemann surface to natural min-max constructions of harmonic maps to high-dimensional spheres. I'll explain how our methods give a new approach to producing extremal metrics for the first and second Laplacian eigenvalues, while yielding new estimates and rigidity results for related shape optimization problems in spectral geometry.

XUWEN ZHU, Northeastern University
Spectral properties of spherical conical metrics

This talk will focus on the recent works on the spectral properties of constant curvature metrics with conical singularities on surfaces. The motivation comes from earlier works joint with Rafe Mazzeo on the study of deformation of such spherical metrics with large cone angles, which suggests that there is a deep connection between the geometric properties of the moduli space and the analytical properties of the associated singular Laplace operator. In this talk I will talk about a joint work with Bin Xu on spectral characterization of the monodromy of such metrics, and work in progress with Mikhail Karpukhin on the relation of spectral properties with harmonic maps.