Recent Advances in Harmonic and Complex Analysis
Développements récents en analyses harmonique et complexe
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LUDOVICK BOUTHAT, Université Laval
The norm of an infinite L-matrix

We know that any linear application from $\mathbb{C}^n$ to $\mathbb{C}^n$ can be described with an $n \times n$ square matrix. The space ℓ^2 of square-summable sequences indexed by the natural numbers is a generalization of $\mathbb{C}^n$ to infinite dimension. We find that the operators, in the case of ℓ^2, can be described by infinite matrices. However, not all infinite matrices gives us an operator on ℓ^2. It is natural to wonder which infinite matrices are a representation of an operator on ℓ^2, and what is their norm. Because of their applications in the problem of the caracterisation of the multipliers in the weighted Dirichlet spaces, we restrict ourselves to the case of infinite L-matrices. An infinite positive L-matrix is an infinite matrix which is defined by a sequence $(a_n)_{n\geq 0}$ of positive real numbers and which is of the form

$$A = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 & \cdots \\ a_1 & a_1 & a_2 & a_3 & \cdots \\ a_2 & a_2 & a_2 & a_3 & \cdots \\ a_3 & a_3 & a_3 & a_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

We will use the Schur test to find some conditions on the sequence $(a_n)_{n\geq 0}$ for A to be an operator on ℓ^2 and to find an upper bound on the ℓ^2 norm of A. Moreover, we will use these tools to find the exact norm of a particular set of L-matrices.

ALEXANDER BRUDNYI, University of Calgary
On nonlinear Runge approximation problems

We present interpolation and approximation results for bounded holomorphic maps into complex manifolds.

ALMAZ BUTAEV, University of Calgary
On geometric preduals of jet spaces on subsets of $\mathbb{R}^n$

For a closed set $S \subset \mathbb{R}^n$ the jet space $J_b^{k,\omega}(S)$ is the Banach space of vector functions whose components are partial derivatives of functions in $C_b^{k,\omega}(\mathbb{R}^n)$ evaluated at points of S equipped with the corresponding quotient norm. The geometric predual $G_J^{k,\omega}(S)$ of $J_b^{k,\omega}(S)$ is the minimal closed subspace of the dual $\left(C_b^{k,\omega}(\mathbb{R}^n)\right)^*$ containing the evaluation functionals of all partial derivatives of order $\leq k$ at points in S. In this talk, we study some geometric properties of spaces $G_J^{k,\omega}(S)$ related to the classical Whitney problems. This talk is based on joint work with Alex Brudnyi.

GALIA DAFNI, Concordia University
Extension domains for bmo

In joint work with Almaz Butaev (Calgary), we consider the problem of characterizing domains $\Omega \subset \mathbb{R}^n$ for which there exists a bounded linear extension operator from $\mathrm{bmo}(\Omega)$ to $\mathrm{bmo}(\mathbb{R}^n)$, where bmo denotes the nonhomogeneous (also called "local") space of functions of bounded mean oscillation, defined by Goldberg. The analogous problem for BMO was solved by Jones, who identified extension domains for BMO with uniform domains. He subsequently defined local versions of these domains, called (ϵ, δ) domains, and proved extension results for Sobolev functions on such domains. We show that the condition on the domain is both necessary and sufficient for the extension of bmo functions.

PAUL GAUTHIER, Université de Montréal

Asymptotic first boundary value problem for holomorphic functions of several complex variables

Theorem (with Mohammad Shirazi, McGill University).
Let M be a complex manifold endowed with a distance d and a regular Borel measure μ, such that non-empty open sets have positive measure. Let $U \subset M$ be an arbitrary Stein domain and $\psi \in \mathcal{M}(\partial U)$ an arbitrary Borel measurable function on the boundary ∂U, whose restriction to some closed subset $S \subset \partial U$ is continuous. Then, for an arbitrary regular σ-finite Borel measure ν on ∂U, there exists a holomorphic function f on U, such that, for ν-almost every $p \in \partial U$, and for every $p \in S$, $f(x) \to \psi(p)$, as $x \to p$ outside a set of μ-density 0 at p relative to U.

RYAN GIBARA, Université Laval

Boundedness and continuity of rearrangements on spaces defined by mean oscillation

In joint work with Almut Burchard and Galia Dafni, we study the boundedness and continuity of rearrangement operators on the space BMO of functions of bounded mean oscillation. New bounds are obtained on the BMO-seminorm of the decreasing and the symmetric decreasing rearrangements, both of which are shown to be discontinuous as maps on BMO. The corresponding questions of boundedness and continuity for both these rearrangements on the space VMO of functions of vanishing mean oscillation is then addressed.

ADI GLUCKSAM, University of Toronto

Computability of harmonic measures

In this talk I will present the new notion of computable harmonic approximation, and show that for an arbitrary domain, computability of the harmonic measure for a single point implies its computability for any point. Nevertheless, different points may require different algorithms, which gives rise to surprisingly natural examples of continuous functions whose values can be computed at any point but cannot be computed using same algorithm on their entire domain. I will present counter examples supporting this and study the conditions under which the harmonic measure is computable uniformly, that is by a single algorithm, and characterize them for regular domains with a computable boundary.

This talk is based on a joint work with I. Binder, C. Rojas, and M. Yampolsky.

WENBO LI, University of Toronto

Conformal dimension and minimality of stochastic objects

In this talk, we discuss the conformal dimension of some stochastic objects. The conformal dimension of a metric space is the infimum of the Hausdorff dimension of all its quasisymmetric images. We call a metric space minimal if its conformal dimension equals its Hausdorff dimension. We begin with a construction of a graph of a random function which is minimal. Inspired by this, we apply the same techniques to the study of 1-dimensional Brownian graphs. The main tool is the Fuglede modulus. This is a joint work with Ilia Binder and Hrant Hakobyan.

JAVAD MASHREGHI, Laval University

Outer Functions and the Schur Class

It is well known that the composition of two inner functions is an inner function. Parallel to this result, the composition of an outer function with a self map of the open unit disk is also outer. The result, while known and classic, is not that trivial. We present an approach via uniform integrability.

This is a joint work With T. Ransford.

FRÉDÉRIC MORNEAU-GUÉRIN, Université TÉLUQ

La $*$-stabilité de l'espace pondéré des suites de carré sommable sur la somme directe de groupes abéliens finis

Au cours de cet exposé, nous présenterons diverses conditions que doit nécessairement satisfaire une fonction de pondération sur un groupe abélien discret afin que l'espace pondéré des suites de carré sommable définies sur ce même groupe soit stable sous le produit de convolution. Nous nous pencherons ensuite plus spécifiquement sur le cas où le groupe sous-jacent est la somme directe de groupes abéliens finis.

PIERRE-OLIVIER PARISÉ, Université Laval

Cesàro summability of Taylor series in weighted Dirichlet spaces

A recent result of J. Mashreghi and T. Ransford has shown that, for a weighted Dirichlet space $\mathcal{D}_\omega$ where $\omega : \mathbb{D} \to (0, \infty)$ is a superharmonic function on the unit disk $\mathbb{D}$, the Cesàro means of order 1 of the partial sums $s_n[f]$ of the Taylor expansion of a function $f \in \mathcal{D}_\omega$ converge to the function in the norm of the space. However, it is known that, for certain weights ω, the partial sums themselves fail to converge. This leads us to the following question : Do the Cesàro means of order $\alpha > 0$ of $s_n[f]$ converge to f in the space $\mathcal{D}_\omega$ for any superharmonic weight ω?

In this talk, I will present the following result for the spaces $\mathcal{D}_\omega$: If $\alpha > \frac{1}{2}$, the Cesàro means of order α always converge to the function f in any space $\mathcal{D}_\omega$, but if $\alpha \leq \frac{1}{2}$, it breaks down for some superharmonic weight ω. This result contrasts with what is known on Cesàro means of order $\alpha > 0$ in the disk algebra and the Hardy space H^1. (Joint work with Javad Mashreghi and Thomas Ransford).

THOMAS RANSFORD, Université Laval

A Gleason-Kahane-Żelazko theorem for reproducing kernel Hilbert spaces.

The Gleason-Kahane-Żelazko theorem states that a linear functional on a Banach algebra that is non-zero on invertible elements is necessarily a scalar multiple of a character. In this talk I shall describe an analogue of this result for a certain class of Hilbert spaces. (Joint work with Cheng Chu, Michael Hartz and Javad Mashreghi.)

LARISSA RICHARDS, University of Toronto

On the rate of convergence of discrete interfaces to SLE.

We will present recent developments in generating a general framework for establishing a rate of convergence of the critical interfaces of various critical lattice models to SLE. Following the work of S. Smirnov and A. Kemppainen and the work of F. Viklund, assuming a polynomial rate of convergence of the martingale observable functions we can obtain a polynomial rate of convergence provided the random curve satisfies some mild conditions. We will discuss the application of this framework to percolation, harmonic explorer, and Ising model.

SCOTT RODNEY, Cape Breton University

Bounded Weak Solutions of Second Order Linear PDEs with Data in Orlicz Spaces

Reporting on joint work with David Cruz-Uribe (UAlabama) and S. Francis MacDonald (CBU math student). For a non-negative definite symmetric matrix valued function $Q = Q(x)$ in a bounded domain $\Omega \subset \mathbb{R}^n$ with $n \geq 3$, we consider weak solutions of Dirichlet problems for linear equations of the form

$$(**) - \frac{1}{v}\mathrm{Div}\left(\sqrt{Q}\nabla u\right) + \mathbf{H}Ru - \frac{1}{v}S'[\mathbf{G}uv] + Fu = f - \frac{1}{v}T'[gv]$$

for $x \in \Omega \ v - a.e.$ Here, the weight $v \in L^1(\Omega)$ satisfies $|Q(x)|_{op} \leq kv(x)$ in Ω where k is a constant. R, S, T are n-tuples of first order vectorfields with adjoints R', S', T'. The data functions f, g, coefficient functions $\mathbf{H}, \mathbf{G}, F$ and are assumed to belong to Orlicz classes associated to the Young functions

$$A(t) = t^{\sigma'} \log(e+t)^q, \ B(t) = A^2(t) \text{ respectively}$$

where $q > \sigma'$, the dual exponent of $\sigma > 1$ that describes the gain in a Sobolev inequality associated to $Q(x)$ and v. Under the assumption of a positivity condition on the vectorfields, we show that any non-negative weak solution u of equation (**) is bounded with

$$\|u\|_{L^\infty(v;\Omega)} \leq C\left(\|u\|_{QH_0^1(\Omega)} + \|f\|_{L^A(\Omega)} + \|g\|_{L^B(\Omega)}\right)$$

where C is independent of f, g, and u.

IGNACIO URIARTE-TUERO, University of Toronto
Two weight norm inequalities for singular integrals in $\mathbb{R}^n$

In this talk I will present the most recent advances in the problem of characterizing the two weight norm inequality for singular integrals in Euclidean space. In particular two weight local Tb theorems in n dimensions with an energy side condition. The talk will be self-contained. Mostly joint work with Grigoriadis, Paparizos, Sawyer, Shen. The talk will be self-contained.

WILLIAM VERREAULT, Université Laval
Nonlinear Oscillatory Expansions of holomorphic functions

In 1995, R. Coifman discovered a nonlinear analogue of Fourier series called Blaschke unwinding series. This iterative Blaschke factorisation has a wide range of practical applications, but it is not well understood. In recent years, the method has been rediscovered by T. Qian et al. and extensively studied, while Coifman and collaborators have studied other unwindings and convergence in given function spaces such as orthogonal decompositions of invariant subspaces of Hardy spaces.

We present results that explain why this Blaschke factorisation only corresponds to a specific (and the most simple) type of unwinding of holomorphic functions, and, using techniques from operator theory, we give necessary and sufficient conditions for the convergence of the unwinding series.

JAMES WILSON, University of Vermont
Discretization of adapted functions

We say that a family $\{\psi_\gamma\}_{\gamma\in\Gamma} \subset L^2(\mathbf{R}^d)$ is almost-orthogonal if there is a finite R so that, for all finite $\mathcal{F} \subset \Gamma$ and all linear combinations $\sum_{\gamma\in\mathcal{F}} \lambda_\gamma\psi_\gamma$, where $\{\lambda_\gamma\}_{\gamma\in\mathcal{F}} \subset \mathbf{C}$,

$$\left\|\sum_{\mathcal{F}} \lambda_\gamma\psi_\gamma\right\|_2 \leq R\left(\sum_{\mathcal{F}} |\lambda_\gamma|^2\right)^{1/2}.$$

The least R for which this holds is called the family's almost-orthogonality constant, denoted $\|\{\psi_\gamma\}_{\gamma\in\Gamma}\|_{AO(\Gamma)}$. The almost-orthogonality constant can be used to quantify how far a family is from having certain useful properties (orthonormality, being a frame, etc.). We show:

Theorem. Let $0 < \alpha, \tau \leq 1$. Let $\{f^{(Q)}\}_{Q\in\mathcal{D}}$ be a family of functions indexed over the dyadic cubes $\mathcal{D}$ in $\mathbf{R}^d$, satisfying: a) $\forall Q \in \mathcal{D}$, supp $f^{(Q)} \subset \overline{Q}$; b) $\forall x, x' \in \mathbf{R}^d(|f^{(Q)}(x) - f^{(Q)}(x')| \leq (|x-x'|/\ell(Q))^\alpha$, where $\ell(Q)$ is Q's sidelength. For each $Q \in \mathcal{D}$ let $\mathcal{G}(Q)$ be a set of disjoint dyadic cubes J such that $\ell(J) \leq \tau\ell(Q)$ and $\cup_{\mathcal{G}(Q)} J = Q$. Set

$$f^{(Q)}_{\mathcal{G}(Q)} := \sum_{J\in\mathcal{G}(Q)} f^{(Q)}_J \chi_J,$$

where $f^{(Q)}_J$ means $f^{(Q)}$'s average over J. There is a constant $C(\alpha, d)$, depending only on α and d, so that

$$\left\|\left\{\frac{f^{(Q)} - f^{(Q)}_{\mathcal{G}(Q)}}{|Q|^{1/2}}\right\}_{Q\in\mathcal{D}}\right\|_{AO(\mathcal{D})} \leq C(\alpha, d)\tau^\alpha,$$

where $|Q| =$ the measure of Q.

What this means is that, if we apply sufficiently fine dyadic stopping times to the functions in $\{f^{(Q)}/|Q|^{1/2}\}_{Q\in\mathcal{D}}$, the resulting family $\{f^{(Q)}_{\mathcal{G}(Q)}/|Q|^{1/2}\}_{Q\in\mathcal{D}}$ is close to $\{f^{(Q)}/|Q|^{1/2}\}_{Q\in\mathcal{D}}$ in the almost-orthogonal sense. If time permits we will say a few words about a companion result (in different ways stronger and weaker) for such families when $\alpha = 1$.

JIE XIAO, Memorial University
An optimal regularity for the planar $\bar{\partial}$-equation

Based on a recent work joint with C. Yuan, this talk shows that given $0 < p < 3$ and a complex Borel measure μ on the unit disk $\mathbb{D}$ the $\bar{\partial}$-equation

$$\partial_{\bar{z}} u(z) = \frac{d\mu(z)}{(2\pi i)^{-1} d\bar{z} \wedge dz} \quad \forall \quad z \in \mathbb{D}$$

has a distributional solution (initially defined on $\overline{\mathbb{D}} = \mathbb{D} \cup \mathbb{T}$ of $\mathbb{D}$) $u \in \mathcal{L}^{2,p}(\mathbb{T})$ (the quadratic Campanato space) if and only if the complex potential

$$\overline{\mathbb{D}} \ni z \mapsto \int_{\mathbb{D}} (1 - z\bar{w})^{-1}\, d\bar{\mu}(w)$$

belongs to $\mathcal{L}^{2,p}(\mathbb{T})$, thereby resolving the Carleson's corona and the Wolff's ideal problems for the algebra $M\big(\mathcal{CA}_p(\mathbb{D})\big)$ of all analytic pointwise multiplications of the analytic version $\mathcal{CA}_p(\mathbb{D})$ of $\mathcal{L}^{2,p}(\mathbb{T})$.

MALIK YOUNSI, University of Hawaii
Holomorphic motions, capacity and conformal welding

The notion of a holomorphic motion was introduced by Mané, Sad and Sullivan in the 1980's, motivated by the observation that Julia sets of rational maps often move holomorphically with holomorphic variations of the parameters. In the years that followed, the study of the behavior of various set-functions under holomorphic motions became an area of significant interest. For instance, holomorphic motions played a central role in the work of Astala on distortion of Hausdorff dimension and area under quasiconformal mappings.

In this talk, I will first review the basic notions and results related to holomorphic motions, including the extended lambda lemma. I will then present some recent results on the behavior of logarithmic capacity and analytic capacity under holomorphic motions. The proofs involve different notions such as conformal welding, quadratic Julia sets and harmonic measure. This is joint work with Tom Ransford and Wen-Hui Ai.