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Bounded Weak Solutions of Second Order Linear PDEs with Data in Orlicz Spaces

Reporting on joint work with David Cruz-Uribe (UAlabama) and S. Francis MacDonald (CBU math student). For a non-negative definite symmetric matrix valued function $Q = Q(x)$ in a bounded domain $\Omega \subset \mathbb{R}^n$ with $n \geq 3$, we consider weak solutions of Dirichlet problems for linear equations of the form

$$(**) - \frac{1}{v}\mathrm{Div}\left(\sqrt{Q}\nabla u\right) + \mathbf{H}Ru - \frac{1}{v}S'[\mathbf{G}uv] + Fu = f - \frac{1}{v}T'[gv]$$

for $x \in \Omega$ $v - a.e.$ Here, the weight $v \in L^1(\Omega)$ satisfies $|Q(x)|_{op} \leq kv(x)$ in Ω where k is a constant. R, S, T are n-tuples of first order vectorfields with adjoints R', S', T'. The data functions f, g, coefficient functions $\mathbf{H}, \mathbf{G}, F$ and are assumed to belong to Orlicz classes associated to the Young functions

$$A(t) = t^{\sigma'}\log(e+t)^q,\ B(t) = A^2(t) \text{ respectively}$$

where $q > \sigma'$, the dual exponent of $\sigma > 1$ that describes the gain in a Sobolev inequality associated to $Q(x)$ and v. Under the assumption of a positivity condition on the vectorfields, we show that any non-negative weak solution u of equation (**) is bounded with

$$\|u\|_{L^\infty(v;\Omega)} \leq C\left(\|u\|_{QH_0^1(\Omega)} + \|f\|_{L^A(\Omega)} + \|g\|_{L^B(\Omega)}\right)$$

where C is independent of f, g, and u.