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An optimal regularity for the planar $\bar{\partial}$-equation

Based on a recent work joint with C. Yuan, this talk shows that given $0 < p < 3$ and a complex Borel measure μ on the unit disk $\mathbb{D}$ the $\bar{\partial}$-equation

$$\partial_{\bar{z}} u(z) = \frac{d\mu(z)}{(2\pi i)^{-1} d\bar{z} \wedge dz} \quad \forall \quad z \in \mathbb{D}$$

has a distributional solution (initially defined on $\overline{\mathbb{D}} = \mathbb{D} \cup \mathbb{T}$ of $\mathbb{D}$) $u \in \mathcal{L}^{2,p}(\mathbb{T})$ (the quadratic Campanato space) if and only if the complex potential

$$\overline{\mathbb{D}} \ni z \mapsto \int_{\mathbb{D}} (1 - z\bar{w})^{-1} \, d\bar{\mu}(w)$$

belongs to $\mathcal{L}^{2,p}(\mathbb{T})$, thereby resolving the Carleson's corona and the Wolff's ideal problems for the algebra $M\big(\mathcal{CA}_p(\mathbb{D})\big)$ of all analytic pointwise multiplications of the analytic version $\mathcal{CA}_p(\mathbb{D})$ of $\mathcal{L}^{2,p}(\mathbb{T})$.