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How concentrated can the divisors of a typical integer be?

The Delta function measures the concentration of the sequence of divisors of an integer. Specifically, given an integer n, we write $\Delta(n)$ for the maximum over y of the number of divisors of n lying in the dyadic interval $[y, 2y]$. It was introduced by Hooley in 1979 because of its connections to various problems in Diophantine equations and approximation. In 1984, Maier and Tenenbaum proved that $\Delta(n) > 1$ for almost all integers n, thus settling a 1948 conjecture due to Erdős. In subsequent work, they proved that $(\log\log n)^{c+o(1)} \le \Delta(n) \le (\log\log n)^{\log 2+o(1)}$, where $c = (\log 2)/\log(\frac{1-1/\log 27}{1-1/\log 3}) \approx 0.33827$ for almost all integers n. In addition, they conjectured that $\Delta(n) = (\log\log n)^{c+o(1)}$ for almost all n. In this talk, I will present joint work with Kevin Ford and Ben Green that disproves the Maier-Tenenbaum conjecture by replacing the constant c in the lower bound by another constant $c' = 0.35332277\ldots$ that we believe is optimal. We also prove analogous results about permutations and polynomials over finite fields by reducing all three cases to an archetypal probabilistic model.