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Moduli spaces for unstable Higgs bundles of rank 2 and their geometry

The moduli space of semistable Higgs bundles is widely studied thanks to its rich geometric structure, in particular as it is an example of a Completely Integrable Hamiltonian System. In this talk we shift our focus from semistable to unstable Higgs bundles, guided by the questions of whether moduli spaces for unstable Higgs bundles can be constructed, and if so whether they admit a similarly rich geometric structure. We will start by considering the case of unstable (twisted) Higgs bundles of rank 2 on the projective line and by showing how moduli spaces can be constructed explicitly in this setting. We will then consider such Higgs bundles on a curve of arbitrary genus and explain how recent results in a generalisation of Geometric Invariant Theory (GIT), called Non-Reductive GIT, can be used to construct moduli spaces for them. We will finish by briefly describing initial steps towards understanding the geometry of these moduli spaces.