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Effective results on the structure of sumsets

Given a finite set $A \subset \mathbb{Z}^d$ with convex hull $\text{conv}(A)$, we have a trivial inclusion between the iterated sumset and the dilated convex hull, namely $NA \subset (N \text{conv}(A)) \cap \mathbb{Z}^d$. But does equality ever hold? In fact there is an easily-described exceptional set $E_N(A)$ for which $E_N(A) \cap NA = \emptyset$, but one may nonetheless ask: does equality hold up to these exceptions?

Granville–Shakan recently showed that, if N is large enough, the answer to this question is yes, equality does hold. However, for all $d \geq 2$ their results gave only an ineffective lower bound on what 'large enough' should mean. In this talk we will describe two new pieces of work on this question: a new bound in the case $d = 1$, which is tight for several infinite families of sets A , and the first effective bounds for arbitrary A when $d \geq 2$. These results are joint work with Granville and with Granville–Shakan respectively. If time permits, we will describe the connections between this work and Khovanskii's theorem (that the size of NA is a polynomial in N , for large enough N).