AARMS-CMS Student Poster Session
Présentations par affiches des étudiants - AARMS-SMC

JOSE ALVILEZ, University of Waterloo

Pathwise integration over rough paths for a model-free formulation of finance

While the classical theory of stochastic integration is elegant and deep, it is pedagogically quite inaccessible and is unable to handle paths that are more irregular than Brownian motion. As a solution, Hans Föllmer formulated a pathwise stochastic calculus, which is able to replicate the formulas of Itô in an elementary way. Föllmer's insight is that a pathwise calculus hinges on the existence of quadratic variation along a sequence of refining partitions.

Studying pathwise quadratic variation—and its extensions—is the focus of this poster presentation. This allows us to obtain a model-free formulation of mathematical finance and extend the current state-of-the-art in volatility modelling established by Gatheral et al. Gatheral's empirical insight is that, across a wide array of assets, volatility processes are "rougher" than Brownian motion and thus not amenable to the theory of stochastic calculus of continuous semimartingales.

This poster will address Gatheral's findings and non-normality of asset processes by presenting the following: (i) A generalisation of the quadratic roughness property recently discovered by Cont and Das to arbitrary p-roughness; (ii) A discussion on the properties of the Takagi-van der Waerden functions, a class of fractal functions that serve as rough integrators, in the context of p-roughness.

Ultimately, the hope is that the pathwise approach helps us present the current state of volatility research in an elementary fashion.

ALI ASADI-VASFI, University of Tehran

The radius of comparison of the crossed product by a tracially strictly approximately inner action

Let G be a finite group, let A be an infinite-dimensional stably finite simple unital C*-algebra, and let $\alpha\colon G \to \mathrm{Aut}(A)$ be a tracially strictly approximately inner action of G on A. Then the radius of comparison satisfies $\mathrm{rc}(A) \leq \mathrm{rc}\left(A \rtimes_\alpha G\right)$ and if $A \rtimes_\alpha G$ is simple, then $\mathrm{rc}(A) \leq \mathrm{rc}\left(A \rtimes_\alpha G\right) \leq \mathrm{rc}(A^\alpha)$.

Also, for every finite group G and for every $\eta \in \left(0, \frac{1}{\mathrm{card}(G)}\right)$, we construct a simple separable unital AH algebra A with stable rank one and a strictly approximately inner action $\alpha\colon G \to \mathrm{Aut}(A)$ such that:

(1) α is pointwise outer and doesn't have the weak tracial Rokhlin property.

(2) $\mathrm{rc}(A) = \mathrm{rc}\left(A \rtimes_\alpha G\right) = \eta$.

MARYAM BASIRI, University of Ottawa

Pushing the boundaries: the existence of solutions for a free boundary problem modeling the spread of ecosystem engineers

The overwhelming majority of models for the spread of an invasive species into a new environment are based on Fisher's reaction-diffusion equation. This approach assumes that habitat quality is independent of the population. Ecosystem engineers are species that modify their environment to make it (more) suitable for them. Beavers are a well-known engineering species. A potentially more appropriate modeling approach is to adapt the well-known Stefan problem of melting ice. Ahead of the front, the habitat is unsuitable for the species (the ice); behind the front, the habitat is suitable (the open water). The engineering action of the population moves the boundary ahead (the melting). This modeling approach leads to a time-dependent free boundary problem where the boundary corresponds to the edge of the population front.

We present a novel model for the spread of ecosystem engineers as a free boundary problem. We derive the semilinear parabolic equation from an individual random walk model. The Stefan condition for the moving boundary is replaced by a biologically derived two-sided condition that models the movement behavior of individuals at the boundary as well as the process by which the population moves the boundary to expand their territory.

We prove the local existence of solutions for the model. We assign a convex functional to this problem so that the evolution system governed by this convex potential is exactly the system of evolution equations describing the above model. We shall then apply variational and fixed-point methods to deal with this free boundary problem.

DAMANVIR SINGH BINNER, Simon Fraser University
Proofs of Berkovich and Uncu's Conjectures on Integer Partitions using Frobenius numbers

We use techniques from elementary number theory (such as Frobenius numbers) to combinatorially prove four recent conjectures of Berkovich and Uncu (Ann. Comb. 23 (2019) 263284) regarding inequalities between the sizes of two closely related sets consisting of integer partitions whose parts lie in the interval s, s+1,..., L+s. Further restrictions are placed on the sets by specifying impermissible parts as well as a minimum part.

BENOÎT CORSINI, McGill University
The height of Mallow trees

Random binary search trees are obtained by recursively inserting the elements $\sigma(1), \sigma(2), \ldots, \sigma(n)$ of a uniformly random permutation σ of $[n] = \{1, \ldots, n\}$ into a binary search tree data structure. Devroye (1986) proved that the height of such trees is asymptotically of order $c^* \log n$, where $c^* = 4.311\ldots$ is the unique solution of $c \log((2e)/c) = 1$ with $c \geq 2$. Here, we study the structure of binary search trees $T_{n,q}$ built from Mallows permutations. A $\mathrm{Mallows}(q)$ permutation is a random permutation of $[n] = \{1, \ldots, n\}$ whose probability is proportional to $q^{\mathrm{Inv}(\sigma)}$, where $\mathrm{Inv}(\sigma) = \#\{i < j : \sigma(i) > \sigma(j)\}$. This model generalizes random binary search trees, since $\mathrm{Mallows}(q)$ permutations with $q = 1$ are uniformly distributed. The laws of $T_{n,q}$ and $T_{n,q^{-1}}$ are related by a simple symmetry (switching the roles of the left and right children), so it suffices to restrict our attention to $q \leq 1$.

We show that, for $q \in [0, 1]$, the height of $T_{n,q}$ is asymptotically $(1 + o(1))(c^* \log n + n(1 - q))$ in probability. This yields three regimes of behaviour for the height of $T_{n,q}$, depending on whether $n(1 - q)/\log n$ tends to zero, tends to infinity, or remains bounded away from zero and infinity. In particular, when $n(1 - q)/\log n$ tends to zero, the height of $T_{n,q}$ is asymptotically of order $c^* \log n$, like it is for random binary search trees. Finally, when $n(1 - q)/\log n$ tends to infinity, we prove stronger tail bounds and distributional limits for the height of $T_{n,q}$.

HERMIE MONTERDE, University of Manitoba
Quantum State Transfer and Strong Cospectrality

A quantum spin network can be modelled by an undirected graph G whose vertices are the qubits in the network, where an edge exists between two interacting qubits. Depending on the dynamics, the evolution of the quantum spin network at time t is given by the matrix $\exp(-itM)$, where M is a Hermitian matrix associated to G. One major problem involving quantum spin networks is determining a time t such that a state of a qubit q_1 is transfered to another qubit q_2 with a particular level of probability. We call this phenomenon quantum state transfer, and the level of probability gives rise to various types of state transfer. In this presentation, we introduce different types of quantum state transfer on graphs, and present known facts. We also discuss the concept of strong cospectrality, a necessary condition for some types of quantum state transfer.

AARON SLOBODIN, The University of Victoria
2-Limited Broadcast Domination in Grid Graphs

Suppose there is a transmitter located at each vertex of a graph G. A k-limited broadcast on G is an assignment of the integers $0, 1, \ldots, k$ to the vertices of G. The integer assigned to the vertex x represents the strength of the broadcast from x, where strength 0 means the transmitter at x is not broadcasting. A broadcast of positive strength s from x is heard by all vertices at distance at most s from x. A k-limited broadcast is called dominating if every vertex assigned 0 is within distance d of a vertex whose transmitter is broadcasting with strength at least d. The k-limited broadcast domination number of G is the minimum possible value of the sum of the strengths of the broadcasts in a k-limited dominating broadcast of G. Observe that the 1-limited broadcast domination number of G equals the domination number of G.

We give tight upper and lower bounds for the 2-limited broadcast domination of Cartesian products of paths. The upper bounds are established by explicit constructions. The methods to obtain the lower bounds utilize the dual of 2-limited broadcast domination, 2-limited multipacking.

NAZANIN ZAKER AND LAURENCE KETCHEMEN TCHOUAGA, University of Ottawa

The effect of movement behavior on population density in fragmented landscapes

Landscape fragmentation arises from human activities and natural causes, and may create abrupt transitions (interfaces) in landscape quality. How landscape fragmentation affects ecosystems diversity and stability depends, among other things, on how individuals move through the landscape. In this work, we focus on the movement behavior at an interface between habitat patches of different quality. Specifically, we study how this individual-level behavior affects the steady state of a density of a diffusing and logistically growing population in two adjacent patches.

We consider a model for population dynamics in a habitat consisting of two homogeneous one-dimensional patches in a coupled ecological reaction diffusion equation. The movement between patches is incorporated into the interface conditions. We establish the existence, uniqueness, and global asymptotic stability of the steady state. Then we explore how the qualitative properties of the steady state depend on movement behavior.

We apply our analysis to a previous result where it was shown that a randomly diffusing population in a continuously varying habitat can exceed the carrying capacity at steady state. We clarify the role of nonrandom movement in this context. In particular, we determine conditions on movement rates and patch preference, so that the steady-state density exceeds the carrying capacity.