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Pathwise integration over rough paths for a model-free formulation of finance

While the classical theory of stochastic integration is elegant and deep, it is pedagogically quite inaccessible and is unable to handle paths that are more irregular than Brownian motion. As a solution, Hans Föllmer formulated a pathwise stochastic calculus, which is able to replicate the formulas of Itô in an elementary way. Föllmer's insight is that a pathwise calculus hinges on the existence of quadratic variation along a sequence of refining partitions.

Studying pathwise quadratic variation—and its extensions—is the focus of this poster presentation. This allows us to obtain a model-free formulation of mathematical finance and extend the current state-of-the-art in volatility modelling established by Gatheral et al. Gatheral's empirical insight is that, across a wide array of assets, volatility processes are "rougher" than Brownian motion and thus not amenable to the theory of stochastic calculus of continuous semimartingales.

This poster will address Gatheral's findings and non-normality of asset processes by presenting the following: (i) A generalisation of the quadratic roughness property recently discovered by Cont and Das to arbitrary p-roughness; (ii) A discussion on the properties of the Takagi-van der Waerden functions, a class of fractal functions that serve as rough integrators, in the context of p-roughness.

Ultimately, the hope is that the pathwise approach helps us present the current state of volatility research in an elementary fashion.