Arithmetic Statistics
Statistique Arithmétique
(Org: **Chantal David** (Concordia), **Matilde Lalin** (UdeM) and/et **Jerry Wang** (Waterloo))

EMILIA ALVAREZ, University of Bristol
Moments of the logarithmic derivative of characteristic polynomials from $SO(N)$ and $USp(2N)$

I will discuss recent work with Nina Snaith on asymptotics of moments of the logarithmic derivative of characteristic polynomials of orthogonal $SO(N)$ and symplectic $USp(2N)$ random matrices, evaluated near the point 1. The leading order behaviour in this regime as N tends to infinity is governed by the likelihood that the matrices in each ensemble have an eigenvalue at or near the point 1. These results follow recent work of Bailey, Bettin, Blower, Conrey, Prokhorov, Rubinstein and Snaith, where they compute these asymptotics in the case of unitary random matrices.

EMMA BAILEY, University of Bristol
Moments of Moments of L-functions

This talk will present results on the 'moments of moments' for the random matrix groups associated with the symmetry classes for families of L-functions (motivated by the work of Keating and Snaith, and Katz and Sarnak). We also formulate the moments of moments of $\zeta(1/2+it)$ which, under a conjecture of Conrey et al., have a multiple contour integral representation. For such a representation, we are able to prove that the moments of moments coincide with the random matrix result. This talk includes work joint with Theo Assiotis and Jon Keating.

MARTIN CECH, Concordia University
Mean values of real Dirichlet characters and double Dirichlet series

We will study the double sum of Jacobi symbols

$$\sum_{n\leq X}\sum_{m\leq Y}\left(\frac{m}{n}\right).$$

An asymptotic formula valid for all values of X, Y was found by Conrey, Farmer and Soundararajan by using Poisson summation and then estimating the sums of Gauss sums that arise in the computation. We will study the sum using Mellin inversion twice and investigating the analytic properties of a double Dirichlet series. This leads to the same asymptotic with an improved error term.

ANTOINE COMEAU-LAPOINTE, Concordia University
One-level density of the family of twists of an elliptic curve over function fields

We fix an elliptic curve $E/\mathbb{F}_q(t)$ and consider the family $\{E\otimes\chi_D\}$ of E twisted by quadratic Dirichlet characters. The one-level density of their L-functions is shown to follow orthogonal symmetry for test functions with Fourier transform supported inside $(-1,1)$. As an application, we obtain an upper bound of 3/2 on the average analytic rank. By splitting the family according to the sign of the functional equation, we obtain that at least 12.5% of the family have rank zero, and at least 37.5% have rank one. The Katz and Sarnak philisophy predicts that those percentages should both be 50% and the average analytic rank should be $1/2$. We finish by computing the one-level density of E twisted by Dirichlet characters of order ℓ coprime to q where we obtain a restriction of $(-1/2,1/2)$ on the support.

LUCILE DEVIN, Chalmers University of Technology and Gothenburg University
Chebyshev's bias and sums of two squares

Studying the secondary terms of the Prime Number Theorem in Arithmetic Progressions, Chebyshev claimed that there are more prime numbers congruent to 3 modulo 4 than to 1 modulo 4. This claim was explained and quantified by Rubinstein and Sarnak. We will see how their framework can be adapted to other questions on the distribution of prime numbers. In particular, we will present a new Chebyshev-like claim : there are "more" prime numbers that can be written as a sum of two squares with the even square larger than the odd square than the other way around.

ANUP DIXIT, Chennai Mathematical Institute
On the classification problem for general Dirichlet series

A typical L-function comes equipped with a functional equation, which gives rise to invariants such as degree and conductor. A natural converse question is whether we can determine the L-function from these invariants. We consider this problem in the context of general Dirichlet series, where we define the degree and conductor using growth conditions. Under certain constraints, we show that there are only finitely many general Dirichlet series with a given degree and conductor.

ALEXANDRA FLOREA, Columbia University
Non-vanishing for cubic L-functions

Chowla conjectured that $L(1/2,\chi)$ never vanishes, for χ any Dirichlet character. Soundararajan showed that more than 87.5% of the values $L(1/2,\chi_d)$, for χ_d a quadratic character, do not vanish. Much less is known about cubic characters. Baier and Young showed that more than $X^{6/7-\epsilon}$ of $L(1/2,\chi)$ are non-vanishing, for χ a primitive, cubic character of conductor of size up to X. I will talk about recent joint work with C. David and M. Lalin, where we show that a positive proportion of these central L-values are non-vanishing in the function field setting. This is achieved by computing the first mollified moment using techniques previously developed by the authors in their work on the first moment of cubic L–functions, and by obtaining a sharp upper bound for the second mollified moment, building on work of Soundararajan, Harper and Lester–Radziwill.

AHMET GULOGLU, Bilkent University
Non-vanishing of Cubic Twists of L-functions

By looking at the one-level density for the family of Hecke L-functions associated with primitive cubic Dirichlet characters defined over the Eisenstein field, we show that a positive proportion of the L-functions associated with a thin subfamily of these characters do not vanish at the central point s = 1/2.

ALIA HAMIEH, University of Northern British Columbia
Mean squares of long Dirichlet polynomials with the divisor function $\tau_2(n)$

In this talk, I report on a joint work with Nathan Ng. We prove an asymptotic formula for mean values of long Dirichlet polynomials with coefficients $\tau_2(n)$. This establishes a special case of a conjecture of Conrey and Gonek (1998) that gives an asymptotic estimate for the mean square of the Dirichlet polynomial associated with the divisor function $\tau_k(n)$. Our asymptotic formula has all lower order terms with a power savings error term.

SEOYOUNG KIM, Queen's University
From the Birch and Swinnerton-Dyer conjecture to Nagao's conjecture

Let E be an elliptic curve over $\mathbb{Q}$ with discriminant Δ_E. For primes p of good reduction, let N_p be the number of points modulo p and write $N_p = p+1-a_p$. In 1965, Birch and Swinnerton-Dyer formulated a conjecture which implies

$$\lim_{x\to\infty} \frac{1}{\log x} \sum_{\substack{p\leq x \\ p\nmid \Delta_E}} \frac{a_p \log p}{p} = -r + \frac{1}{2},$$

where r is the order of the zero of the L-function $L_E(s)$ of E at $s = 1$, which is predicted to be the Mordell-Weil rank of $E(\mathbb{Q})$. We show that if the above limit exits, then the limit equals $-r + 1/2$. We also relate this to Nagao's conjecture. This is a recent joint work with M. Ram Murty.

WANLIN LI, MIT
The Central Value of Dirichlet L-functions over Rational Function Fields

The central value of a Dirichlet L-function over $\mathbb{F}_q(t)$ is governed by the Zeta function of a smooth project curve over $\mathbb{F}_q$. Using this connection to geometry, we show a lower bound on the number of quadratic characters with conductor $\leq X$ whose L-functions vanish at the central point. The existence of infinitely many such characters is in contrast with the situation over the rational numbers, where a conjecture of Chowla predicts there should be no such L-functions. Towards this direction, for each fixed q, we give an explicit upper bound on the number of such quadratic characters. This upper bound decreases as q grows and it goes to 0% as $q \to \infty$. In this talk, I will also discuss Dirichlet characters of odd prime order ℓ and the central value of their L-functions. Some of the results in this talk are joint work with Jordan Ellenberg and Mark Shusterman.

ALLYSA LUMLEY, Centre de Reserches Mathématiques
Primes in short intervals: Heuristics and calculations

We formulate, using heuristic reasoning, precise conjectures for the range of the number of primes in intervals of length y around x, where $y \ll (\log x)^2$. In particular, we conjecture that the maximum grows surprisingly slowly as y ranges from $\log x$ to $(\log x)^2$. We will show that our conjectures are somewhat supported by available data, though not so well that there may not be room for some modification. This is joint work with Andrew Granville.

AMITA MALIK, American Institute of Mathematics
Bias statistics for the zeros of L-functions

In this talk, we discuss the arithmetic statistics for the bias density function concerning the distribution of the fractional part of the zeros of L-functions. This bias was first noted by Rademacher in 1956 in the case of the Riemann zeta function and further elucidated by Ford-Zaharescu and Ford-Soundararajan-Zaharescu more recently.

NEHA PRABHU, Indian Institute of Science Education and Research-Pune, India
A joint distribution theorem with applications to extremal primes for elliptic curves

An extremal prime p for an elliptic curve E is one for which $|a_p(E)| = [2\sqrt{p}]$ i.e., $a_p(E)$ is maximal or minimal in view of the Hasse bound. Although an asymptotic for the number of extremal primes up to x for a fixed non-CM elliptic curve seems out of reach, upper bounds have been proved recently. In this talk, assuming GRH, we present a joint distribution result involving the Chebotarev density theorem. As a consequence, we obtain an upper bound for the number of primes satisfying $a_p(E) = [2\sqrt{p}] \mod \ell$ for a sufficiently large prime ℓ. This is joint work with Amita Malik.

BRAD RODGERS, Queen's University
Primes in short intervals in number fields

In this talk I hope to discuss different analogies for the notion of a short interval in algebraic number fields, and in particular discuss a classical conjecture of Goldston and Montgomery about the distribution of primes in short intervals in this more general setting. As we will see for some notions of a short interval their conjecture appears to carry over naturally, while for other notions it appears to not. This is based off the computation of sums of singular series in this setting. This is joint work with Vivian Kuperberg and Edva Roditty-Gershon.

WILL SAWIN, Columbia University
Measures from moments for random groups

In probability theory, it is useful to prove that a given measure is determined by its moments. In arithmetic statistics, we often want a result like this for a measure on groups, e.g. the Cohen-Lenstra measure on finite abelian ℓ-groups (which predicts the distribution of the ℓ-part of the class groups of imaginary quadratic fields). In this setting, "moments" are the expected number of surjections from a random group to a fixed group. I present a new approach to proving a measure is determined by its moments that works even for non-abelian groups, and is applicable in particular to the measure Liu, Wood, and Zureick-Brown used to predict the distribution of the Galois groups of the maximal unramified extension of a random number field.

ARUL SHANKAR, University of
The 2-torsion subgroups of the class groups in families of cubic fields

The Cohen–Lenstra–Martinet conjectures have been verified in only two cases. Davenport–Heilbronn compute the average size of the 3-torsion subgroups in the class group of quadratic fields and Bhargava computes the average size of the 2-torsion subgroups in the class groups of cubic fields. The values computed in the above two results are remarkably stable. In particular, work of Bhargava–Varma shows that they do not change if one instead averages over the family of quadratic or cubic fields satisfying any finite set of splitting conditions.

However for certain "thin" families of cubic fields, namely, families of monogenic and n-monogenic cubic fields, the story is very different. In this talk, we will determine the average size of the 2-torsion subgroups of the class groups of fields in these thin families. Surprisingly, these values differ from the Cohen–Lenstra–Martinet predictions! We will also provide an explanation for this difference in terms of the Tamagawa numbers of naturally arising reductive groups. This is joint work with Manjul Bhargava and Jon Hanke.

QUANLI SHEN, University of Lethbridge
The fourth moment of quadratic Dirichlet L-functions

In 2010, Soundararajan and Young established the asymptotic formula for the second moment of quadratic twists of a cusp form under the generalized Riemann hypothesis (GRH). They also unconditionally obtained the sharp lower bound which matches Keating-Snaith's conjecture. In this talk, I will discuss the fourth moment of quadratic Dirichlet L-functions. Largely based on Soundararajan and Young's work, we obtained the asymptotic formula under GRH, and also the sharp lower bound unconditionally.

STANLEY XIAO, University of Toronto
The number of quartic-D_4 fields having monogenic cubic resolvent ordered by conductor

In this talk we discuss how to count quartic fields whose Galois group is isomorphic to the dihedral group D_4 and whose ring of integers has a monogenic cubic resolvent ring, ordered by their Artin conductor. In particular we give an asymptotic formula for the number of such fields having a given signature. The techniques we develop also enable us to count such quartic fields by discriminant (but we do not obtain an asymptotic formula) and also elliptic curves with a marked 2-torsion point by discriminant. This is joint work with Cindy Tsang.

ASIF ZAMAN, University of Toronto
An approximate form of Artin's holomorphy conjecture and nonvanishing of Artin L-functions

Let k be a number field and G be a finite group, and let $\mathfrak{F}_k^G$ be a family of number fields K such that K/k is normal with Galois group isomorphic to G. Together with Robert Lemke Oliver and Jesse Thorner, we prove for many families that for almost all $K \in \mathfrak{F}_k^G$, all of the L-functions associated to Artin representations whose kernel does not contain a fixed normal subgroup are holomorphic and non-vanishing in a wide region.

I will discuss these results and some of their arithmetic applications. For example, we prove a strong effective prime ideal theorem that holds for almost all fields in several natural large degree families, including the family of degree n S_n-extensions for any $n \geq 2$ and the family of prime degree p extensions (with any Galois structure) for any prime $p \geq 2$. Other applications relate to bounds on ℓ-torsion subgroups of class groups, the extremal order of class numbers, and the subconvexity problem for Dedekind zeta functions.