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From the Birch and Swinnerton-Dyer conjecture to Nagao's conjecture

Let E be an elliptic curve over $\mathbb{Q}$ with discriminant Δ_E. For primes p of good reduction, let N_p be the number of points modulo p and write $N_p = p + 1 - a_p$. In 1965, Birch and Swinnerton-Dyer formulated a conjecture which implies

$$\lim_{x\to\infty} \frac{1}{\log x} \sum_{\substack{p\leq x \\ p\nmid\Delta_E}} \frac{a_p \log p}{p} = -r + \frac{1}{2},$$

where r is the order of the zero of the L-function $L_E(s)$ of E at $s = 1$, which is predicted to be the Mordell-Weil rank of $E(\mathbb{Q})$. We show that if the above limit exits, then the limit equals $-r + 1/2$. We also relate this to Nagao's conjecture. This is a recent joint work with M. Ram Murty.