Additive Combinatorics and Discrete Geometry
Combinatoire Additive et Géométrie Discrète
(Org: **Malabika Pramanik** and/et **Josh Zahl** (UBC))

DANIEL DI BENEDETTO, The University of British Columbia
Discretised point-line incidences and the dimension of Besicovitch sets

We discuss a discretised version of the Szemerédi—Trotter theorem and its application to upper Minkowski dimension estimates for Besicovitch sets in $\mathbb{R}^3$. This talk is based on ongoing joint work with Joshua Zahl.

BRANDON HANSON, UGA
A better-than-Plunnecke bound for $A + 2A$

If A is a finite set in an abelian group, we can measure the additive structure of A by the size of its doubling constant, $K = |A + A|/|A|$. Plunnecke's inequality lets us measure the size of iterated sumsets in terms of K, and in particular it tells us that $|A + A + A| \leq K^3|A|$. The set $A + 2A = \{a + b + b : a, b \in A\}$ is a subset of $A + A + A$ and so the upper bound $K^3|A|$ applies. In this talk, I will describe recent work with G. Petridis where we prove that in fact $|A + 2A| \leq K^{2.95}|A|$, answering a question of B. Bukh.

WEIKUN HE, Korea Institute for Advanced Study
Sum-product in representations of Lie groups

I will present some results in the spirit of Bourgain's discretized sum-product theorem, but in the context of Lie groups and their linear representations.

Based on a joint work with Nicolas de Saxcé.

JONGCHON KIM, University of British Columbia
Estimates for some geometric maximal functions associated with a set of directions

We will discuss geometric maximal functions associated with averages over line segments oriented in a set of directions and their singular integral analogues. The maximal functions can be regarded as "singular" variants of the Nikodym maximal function associated with thin tubes. The main problem is to quantify the dependence of the operator norm on the number and the distribution of directions. We will discuss a divide-and-conquer type approach to this problem for L^2 estimates.

ORIT RAZ, The Hebrew University of Jerusalem
Dimension-expanding polynomials and the discretized Elekes-Rónyai theorem

I will present a recent result, joint with Josh Zahl, asserting that most real bivariate polynomials are “dimension expanding" when applied to a Cartesian product. More concretely, if P is a polynomial that is not of the form $P(x, y) = h(a(x) + b(y))$ or $P(x, y) = h(a(x)b(y))$, then whenever A and B are Borel subsets of $\mathbb{R}$ with Hausdorff dimension $0 < \alpha < 1$, we have that $P(A, B)$ has Hausdorff dimension at least $\alpha + \varepsilon$ for some $\varepsilon(\alpha) > 0$ that is independent of P. This is an analogue of Elekes-Rónyai theorem, which is concerned with the cardinality of $P(A \times B)$ for finite sets A, B.

SOPHIE STEVENS, Radon Institute for Computational and Applied Mathematics (RICAM), Linz
The Elekes-Szabó Problem and the Uniformity Conjecture

The Elekes-Szabó problem is to find an upper bound for $|Z(F) \cap (A \times B \times C)|$ for a 'non-degenerate' trivariate polynomial $F \in \mathbb{R}[x, y, z]$. Here, $Z(F)$ is the zero set of F. If we assume the Uniformity Conjecture, then we show how to obtain stronger bounds for a special family of polynomials in $\mathbb{Q}[x, y, z]$. Our conditional results are quantitatively stronger than the unconditional results of Raz, Sharir and de Zeeuw. In this talk, I will give several applications to additive combinatorics and discrete geometry. For example, to expanders, additive energy bounds, and pinned distances. This is joint work with M. Makhul, O. Roche-Newton and A. Warren.

CAROLINE TERRY, The Ohio State University
A stable arithmetic regularity lemma in finite abelian groups

The arithmetic regularity lemma for $\mathbb{F}_p^n$ (first proved by Green in 2005) states that given $A \subseteq \mathbb{F}_p^n$, there exists $H \leq \mathbb{F}_p^n$ of bounded index such that A is Fourier-uniform with respect to almost all cosets of H. In general, the growth of the index of H is required to be of tower type depending on the degree of uniformity, and must also allow for a small number of non-uniform elements. Previously, in joint work with Wolf, we showed that under a natural model theoretic assumption, called stability, the bad bounds and non-uniform elements are not necessary. In this talk, we present results extending this work to stable subsets of arbitrary finite abelian groups. This is joint work with Julia Wolf.

JONATHAN TIDOR, Massachusetts Institute of Technology
Joints of Varieties

The joints theorem of Guth and Katz states that n lines in $\mathbb{R}^3$ form at most $O(n^{3/2})$ joints, where a joint is the intersection point of 3 non-coplanar lines. The proof of this result introduced a number of techniques that are now part of the standard toolkit of the polynomial method. We generalize this result from lines to varieties. One special case of our result states that n planes (2-flats) in $\mathbb{F}^6$ form at most $O(n^{3/2})$ joints, where a joint is the intersection point of 3 planes that do not all lie in a single hyperplane. Our results introduce new techniques for applying the polynomial method to higher-dimensional objects.

Joint work with Hung-Hsun Hans Yu and Yufei Zhao.

TONGOU YANG, University of British Columbia
Uniform decoupling in l2 for polynomials

For each positive integer d, we prove a uniform l2-decoupling inequality for the collection of all polynomials phases of degree at most d. Our result is intimately related to MR4078083, but we use a different partition that is determined by the geometry of each individual function.

ALEXIA YAVICOLI, University of St Andrews
Patterns in thick compact sets

I will discuss the connection between thickness, winning sets and patterns in compact sets.