
Stochastic Control Methods in Finance and Economics
Méthode de contrôle stochastique en Finances et en Économie
(Org: **Abel Cadenillas** (Alberta) and/et **Bin Zou** (University of Connecticut))

ABEL CADENILLAS, University of Alberta
The Optimal Control of Government Stabilization Funds

We study the optimal control of a government stabilization fund, which is a mechanism to save money in the good economic times to be used in the bad economic times. The objective is to keep the fund as close as possible to a predetermined target. Thus, we consider a running cost associated with the difference between the actual fiscal fund and the fund target. The fund manager exerts control over the fund by making deposits in or withdrawals from the fund. The withdrawals are used to pay public debt or to finance government programs. For the first time in the literature, we find the optimal band for the government stabilization fund. Our results are of interest to practitioners.

RYAN DONNELLY, King's College London
Effort Expenditure for Cash Flow in a Mean-Field Equilibrium

We study a mean-field game framework in which agents expend costly effort in order to transition into a state where they receive cash flows. As more agents transition into the cash flow receiving state, the magnitude of all remaining cash flows decreases, introducing an element of competition whereby agents are rewarded for transitioning earlier. An equilibrium is reached if the optimal expenditure of effort produces a transition intensity which is equal to the flow rate at which the continuous population enters the receiving state. We give closed-form expressions which yield equilibrium when the cash flow horizon is infinite or exponentially distributed. When the cash flow horizon is finite we implement an algorithm which yields equilibrium if it converges. We show that in some cases a higher cost of effort results in the agents placing greater value on the potential cash flows in equilibrium. We also present cases where algorithm fails to converge to an equilibrium.

Joint work with Tim Leung.

DENA FIROOZI, University of Toronto
Electronic Trading, Reinforcement Learning, and Mean-Field Games

We consider electronic markets with many participants trading a common asset that is impacted by all participants' actions and propose a batch reinforcement learning approach to estimate the price dynamics, the impact of trading, and the resulting equilibria. The price model and the impact resulting from trading is updated between epochs of active trading. During epochs of active trading, we use a mean-field game (MFG) framework to approximate the equilibria and obtain optimal trading strategies. To do so, we consider parameterized stochastic linear dynamics together with an entropy-regularized linear-quadratic reward function for each trader. Traders do not specify their trading action, but rather the distribution of their trading action. We obtain exploratory dynamics and rewards capturing repetitive learning under exploration – in the sense of relaxed controls – and show that (i) the optimal distribution of trading action for balancing exploration and exploitation is Gaussian, (ii) the exploitation is captured by the mean of the Gaussian distribution, which is influenced by the mean-field and coincides with the classical results for linear-quadratic MFG systems, (iii) the exploration is captured by the variance of the Gaussian distribution. In this talk, we present our ongoing work.

CHRISTOPH FREI, University of Alberta
Digital Currencies: the Tradeoff between Efficiency and Trust

Minimizing volatility in their exchange rates, new forms of digital currencies become viable alternatives to traditional currencies for payments. We develop a model to capture incentives and preferences behind the use of different payment methods in

over-the-counter market transactions. Our model features the fundamental tradeoff between (i) welfare gains from lower transaction costs and (ii) costs of setting up the digital technology and trust concerns when using digital currencies. While participants in the digital currency benefit from welfare gains, people lacking trust in the digital currency suffer from fewer trading opportunities and decreased welfare when digital currencies become widely used. The talk is based on joint work with Qianhong Huang (University of Alberta).

PETER LAKNER, New York University
High Frequency Asymptotics for the Limit Order Book

We study the one-sided limit order book for sell (or buy) orders and model it as a measure-valued process. Limit sell (or buy) orders are offers to sell (or buy) an equity at a price determined by the seller (or buyer). Market buy (or sell) orders are orders to buy (or sell) an equity at the best, that is, least expensive (most expensive, in case of sell market orders) price offered by previous limit sell (or buy) orders. Limit and market orders arrive to the book according to two independent Poisson processes, and limit orders are placed on the book according to a distribution which varies depending on the current best price. We consider the order book in a high frequency regime in which the rates of incoming limit and market orders are large, and traders place their limit sell orders close to the current best price. We provide weak limits for the price process, the scaled volume, and the scaled measure-valued order book process in the high frequency regime. We then provide an analysis of the long-run behavior of the limiting price process.

ANNE MACKAY, UQAM
Constrained portfolio optimization in variable annuities

We discuss a portfolio management problem in which a rational policyholder of a variable annuity (VA) with minimum accumulation guarantee wants to maximize the utility of her terminal wealth. We assume that the investment mix in the VA account can be modified throughout the contract, subject to a fair pricing constraint. This problem is formulated in terms of constrained optimal stochastic control which requires the maximization of a non-concave utility function, and is solved using a martingale approach. Numerical examples are used to analyze the interplay between the VA fee structure and optimal investment strategies. In particular, we show that there exist different ways to set the VA guarantee fee, and that lower fees can lead to a higher expected utility without increasing the VA provider's liability.

OLEKSII MOSTOVYI, University of Connecticut
Stability of the Indirect Utility Process

We investigate the dynamic stability of the indirect utility process associated with a (possibly suboptimal) trading strategy under perturbations of the market. Establishing biconjugacy characterizations first, we prove continuity and first-order convergence of the indirect-utility process under simultaneous perturbations of the finite variation and martingale parts of the stock price return. We discuss the relation to forward performance process (FPP), and therefore the results of this paper provide an approach for the stability analysis of the FPPs under perturbations of the market.

JINNAO QIU, University of Calgary
Optimal Liquidation in Target Zone Models and Neumann Problem with Singular Terminal Condition

We shall study the optimal liquidation problems in target zone models using dynamic programming methods. Such control problems allow for stochastic differential equations with reflections and random coefficients. The value function is characterized by a Neumann problem of backward stochastic partial differential equations (BSPDEs) with singular terminal conditions. The existence and the uniqueness of strong solution to such BSPDEs are addressed, which in turn yields the optimal feedback control. In addition, the unique existence of strong solution to Neumann problem of general semilinear BSPDEs in finer functions space, a comparison theorem, and a new link between forward-backward stochastic differential equations and BSPDE are proved as well. This is based on joint work with Dr. Robert Elliott and Dr. Wenning Wei.

ALEXANDER SCHIED, University of Waterloo

A central bank strategy for defending a currency target zone

We consider a central bank strategy for maintaining a two-sided currency target zone, in which an exchange rate of two currencies is forced to stay between two thresholds. To keep the exchange rate from breaking the prescribed barriers, the central bank is generating permanent price impact and thereby accumulating inventory in the foreign currency. Historical examples of failed target zones illustrate that this inventory can become problematic, in particular when there is an adverse macroeconomic trend in the market. We model this situation through a continuous-time market impact model of Almgren–Chriss-type with drift, in which the exchange rate is a diffusion process controlled by the price impact of the central bank’s intervention strategy. The objective of the central bank is to enforce the target zone through a strategy that minimizes the inventory in foreign currency. We formulate this objective as a stochastic control problem with infinite time horizon. It is solved by reduction to a singular boundary value problem that was solved by Lasry and Lions (1989). Finally, we provide numerical simulations of optimally controlled exchange rate processes and the corresponding evolution of the central bank inventory. Joint work with Eyal Neuman, Chengguo Weng, and Xiaole Xue.