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**Modeling Population Dynamics: Applications and Recent Developments**  
**Modélisation de dynamiques de populations: applications et développements récents**  
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**JACQUES BELAIR**, Université de Montréal

*Dynamics of a quail population with season-dependent reproduction rates*

We present a model, inspired by field data, for the semi-annual evolution of a population of quails with weather-dependent reproduction rates for each of the intermediate seasons, the density of cover making each of the rates monotone decreasing and bounded away from zero. The ensuing model takes the form of a system of two nonlinear finite-difference equations which was derived previously (Milton and Bélair, *Theoretical Population Biology* [37]) but considered then under considerably simplifying assumptions. A geometric method (generalized cobwebbing) is presented to explicitly determine the asymptotic behaviour of the solutions, and numerical as well as analytical methods are used to establish stability, and bifurcations, of equilibria.

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**YUMING CHEN**, Wilfrid Laurier University

*Dirichlet problem for a diffusive logistic population model with two delays*

We investigate a diffusive logistic equation with non-zero boundary Dirichlet condition and two delays. We first exclude the existence of positive heterogeneous steady states, which implies the uniqueness of constant positive steady states. Then, we analyze the local stability and local Hopf bifurcation at the unique constant positive steady state. We show that multiple delays can induce multiple stability switches. Furthermore, we prove global stability of the positive steady state under certain conditions and obtain global Hopf bifurcation results. Numerical simulations have been carried out to illustrate the obtained theoretical results. This is a joint work with Xuejun Pan and Hongying Shu.

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**HARRY GAEBLER**, University of Guelph

*Modeling Uranium Bioreduction by Multispecies Biofilms*

In oxidized groundwater, soluble uranium ( $U(VI)$ ) has the potential to rapidly disperse and lead to wide-spread contamination. In this work, we develop and discuss two models to investigate the use of in-situ microorganisms, in the form of bacterial biofilms, as an effective treatment technology for uranium contaminated soil and groundwater.

The first model is a reactive transport model that describes uranium bioreduction in porous media. This model is a multiscale model developed first by describing microbial processes at the mesoscale, then upscaling these processes to the macroscale (reactor scale). Processes included in the mesoscale model are hydrodynamics and transport of substrates in the reactor, two biofilm and suspended bacteria species and their respective growth kinetics in the pore space through consumption of appropriate substrates coupled with the bioreduction of uranium, attachment of suspended cells to each of the biofilms, detachment of biofilm cells, and cell lysis.

The second model is a traditional chemostat model of the system described above, which is used to investigate the importance of specific reaction kinetics on the overall effectiveness of uranium bioreduction.

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**TING-HAO HSU**, University of Miami

*Slow-Fast Systems and Regime Shifts*

In ecology, regime shifts are continual rapid change between different long-lasting dynamics. An example is disease outbreak, where a system exhibits qualitative changes after long periods of apparent quiescence. Another example is rapid eco-evolutionary dynamics, which have been observed widely both in predators and in prey. For systems modeling those examples, we derive conditions under which regime shifts occur. Our approach was based on extending the so-called entry-exit function to multi-dimensional slow-fast systems using geometric singular perturbation theory. This is joint work with Shigui Ruan and Gail Wolkowicz.

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**HAO KANG**, University of Miami

*Nonlinear age-structured population models with nonlocal diffusion and nonlocal boundary conditions*

In this paper, we develop the basic theory for age-structured population models with nonlocal diffusion and nonlocal boundary conditions. We first apply the theory of integrated semigroups and non-densely defined operators to the linear equations, study the spectrum, and analyze the asymptotic behavior via asynchronous exponential growth. Then we consider a semilinear equation and a nonlinear equation and establish the existence and stability of the nontrivial equilibria.

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**KUNQUAN LAN**, Ryerson University

*Parabolic boundary value problems and applications to population models with spatially varying growth rate and external forcing*

This presentation is based on recent work on parabolic boundary value problems with nonlinearity changing signs and applications to population models with spatially varying growth rate and external forcing. The fixed point index theory for  $r$ -nowhere normal-outward compact maps obtained by Yang and Lan in 2016 will be presented. This index theory is a generalization of the classical fixed point index theory for compact self-maps defined in cones in Banach spaces, and is a good tool to deal with the above boundary value problems and population models.

This is joint work with Professor Wei Lin at Fudan University

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**ZHENYANG LI**, Honghe University

*DYNAMICS OF MAPS WITH MEMORY*

We consider a map of the unit square which is not 1-1, defined as follows:  $x_{n+1} = M_\alpha(x_{n-1}, x_n) = \tau(\alpha \cdot x_n + (1 - \alpha) \cdot x_{n-1})$ , where  $\tau$  is a one-dimensional map on  $I = [0, 1]$  and  $0 < \alpha < 1$  determines how much memory is being used. We let  $\tau$  to be the symmetric tent map. To study the dynamics of  $M_\alpha$ , we consider the two-dimensional map

$$G_\alpha : [x_{n-1}, x_n] \mapsto [x_n, \tau(\alpha \cdot x_n + (1 - \alpha) \cdot x_{n-1})].$$

The map  $G_\alpha$  for  $\alpha \in (0, 3/4]$  studied in our paper is shown to have various properties depending on the  $\alpha$  applying Tsujii's result. For  $\alpha \in (3/4, 1)$  the map  $G_\alpha$  admits a singular Sinai-Ruelle-Bowen measure in another paper, we do this by applying Rychlik's results for the Lozi map. Unlike the Lozi map, the maps  $G_\alpha$  are not invertible.

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**CHIU-JU LIN**,

*A Novel Delayed Logistic Equations with Decay-Consistent Delay in Growth*

We derive an alternative expression for a delayed logistic equation, assuming that the rate of change of the population depends on three components: growth, death, and intraspecific competition, with the delay in the growth component. Following Arino et al. (J Theor Biol 241(1):109–119, 2006), in our formulation we incorporate the delay in the growth term in a manner consistent with the rate of instantaneous decline in the population. For our new model, we provide a complete global analysis showing that no sustained oscillations are possible. We further extend our formulation to a two species competition and use adaptive dynamics to conclude that, assuming certain trade-offs between the growth rate and the delay, the evolutionary trend is to make the delay approaching an optimal value.

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**WEI LIN**, Fudan, China

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**XINZHI LIU**, University of Waterloo

*Bio-Inspired Coordination and Consensus of Multi-Agent Systems*

Many biological systems, such as swarms of birds, flocks of beasts, schools of fish, armies of ants, and colonies of bees, exhibit fascinating collective behavior. It is observed that in such systems, each individual acts as an autonomous agent and interacts only with its nearby neighbors, while the entire group displays coordinated behavior and can accomplish very complex tasks. Inspired by such collective intelligence of animal groups in nature, there has been an increased research interests in multi-agent systems around the world in recent years. This talk discusses the distributed consensus problems of multi-agent systems with both fixed and switching topologies. Hybrid consensus protocols are proposed to take into consideration of continuous communications among agents and intermittent information exchanges on a sequence of discrete times. Based on the proposed algorithms, the multi-agent systems with the hybrid consensus protocols achieve consensus by employing results from matrix theory and algebraic graph theory. Our results show that the hybrid consensus protocols can solve the consensus problem if the union of continuous-time and discrete-time interaction digraphs contains a spanning tree frequently enough. Simulations are provided to demonstrate the effectiveness of the proposed consensus protocols.

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**ROBERT MCCANN**, University of Toronto

*When do interacting organisms gravitate to the vertices of a regular simplex?*

Flocking and swarming models often assume that organisms interact through a force which is attractive over large distances yet repulsive at short distances. Assume this force is given as a difference of power laws and normalized so that its unique minimum occurs at unit separation. For a range of exponents corresponding to mild repulsion and strong attraction, we show that the minimum energy configuration is uniquely attained — apart from translations and rotations — by equidistributing the organisms over the vertices of a regular top-dimensional simplex (i.e. an equilateral triangle in two dimensions and regular tetrahedron in three).

If the attraction is not assumed to be strong, we show these configurations are at least local energy minimizers in the relevant  $d_\infty$  metric from optimal transportation, as are all of the other uncountably many unbalanced configurations with the same support. We infer the existence of phase transitions.

One ingredient from the proof will be described in another session, namely the establishment of a simple isodiametric variance bound which characterizes regular simplices: it shows that among probability measures on  $\mathbf{R}^n$  whose supports have at most unit diameter, the variance around the mean is maximized precisely by those measures which assign mass  $1/(n + 1)$  to each vertex of a (unit-diameter) regular simplex.

Based on preprint with Tongseok Lim at <https://arxiv.org/abs/1907.13593>

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**SHIGUI RUAN**, University of Miami

*On a Network Model of Two Competitors with Applications to the Invasion and Competition of Aedes Albopictus and Aedes Aegypti Mo*

The two prominent mosquito species, *Aedes aegypti* and *Ae. albopictus*, are the primary vectors that transmit several arboviral diseases, including chikungunya, dengue fever, yellow fever, and Zika. The world is presently experiencing a series of major outbreaks of these vector-borne diseases, so it is very important and necessary to understand the current distributions and movements of these mosquito vectors for successful surveillance and control programs. Based on the invasion of the *Ae. albopictus* mosquitoes and the competition between *Ae. Albopictus* and *Ae. aegypti* mosquitoes in the US, we study a two-species competition model in a network, that is with discrete Laplacian diffusion. In the case of strong-weak competition where the invasive competitor is stronger than the local one, it is shown that the invasive species (*Ae. albopictus*) wins over the local species (*Ae. aegypti*) and the solutions converge uniformly to the semi-positive equilibrium such that the invasive species survives while the local species becomes extinct, and vice versa. In the case of weak-weak competition, the solutions converge uniformly to the positive equilibrium so that both invasive and local species coexist. By using numerical simulations, we apply the two-species competition model in a network to explain the invasion and competition of *Ae. Albopictus* and *Ae. Aegypti* in the US. We also show that discrete Laplacian diffusion induces different spreading speeds in different invasive directions.

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**GUNOG SEO**, Colgate University

*A Predator-Prey Model with a Holling Type I Functional Response Including a Predator Mutual Interference*

The most widely used functional response in describing predator-prey relationships is the Holling type II functional response, where per capita predation is a smooth, increasing, and saturating function of prey density. Beddington and DeAngelis modified the Holling type II response to include interference of predators that increases with predator density. In this talk, I introduce a predator-interference term into a Holling type I functional response. I explain the ecological rationale for the response and note that the phase plane configuration of the predator and prey isoclines differs greatly from that of the Beddington-DeAngelis response; for example, in having three possible interior equilibria rather than one. In fact, this new functional response seems to be quite unique. I use analytical and numerical methods to show that the resulting system shows a much richer dynamical behavior than the Beddington-DeAngelis response, or other typically used functional responses. For example, cyclic-fold, saddle-fold, homoclinic saddle connection, and multiple crossing bifurcations can all occur. I then use a smooth approximation to the Holling type I functional response with predator mutual interference to show that these dynamical properties do not result from the lack of smoothness, but rather from subtle differences in the functional responses. This is joint work with Donald L. DeAngelis at the University of Miami.

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**ZHONGWEI SHEN**, University of Alberta

*Transient dynamics in stochastic population models*

Transient dynamics, often observed in multi-scale systems, are roughly defined to be the interesting dynamical behaviours that display over finite time periods. For a class of randomly perturbed dynamical systems that arise in chemical reactions and population dynamics, and that exhibit persistence dynamics over finite time periods and extinction dynamics in the long run, we use quasi-stationary distributions (QSDs) to rigorously capture the transient states governing the transient dynamics. We study the noise-vanishing concentration of the QSDs to gain information about the transient states.

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**JUNPING SHI**, College of William & Mary

*Spectral Monotonicity of Perturbed Quasi-positive Matrices with Applications in Population Dynamics*

The reduction principle for structured population model states that fast dispersal will reduce the growth rate for a spatially distributed population. This reduction principle is proved in two different ways: one using graph theoretical method, and the one using analytic method. Applications of the reduction principle are shown for scalar model, competition model and predator-prey model. This is a joint work with Shanshan Chen, Zhisheng Shuai and Yixiang Wu.

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**ROBERT SMITH?**, The University of Ottawa

*Using non-smooth models to determine thresholds for microbial pest management*

Releasing infectious pests could successfully control and eventually maintain the number of pests below a threshold level. To address this from a mathematical point of view, two non-smooth microbial pest-management models with threshold policy are proposed and investigated in the present paper. First, we establish an impulsive model with state-dependent control to describe the cultural control strategies, including releasing infectious pests and spraying chemical pesticide. We examine the existence and stability of an order-1 periodic solution, the existence of order-k periodic solutions and chaotic phenomena of this model by analyzing the properties of the Poincaré map. Secondly, we establish and analyze a Filippov model. By examining the sliding dynamics, we investigate the global stability of both the pseudo-equilibria and regular equilibria. The findings suggest that we can choose appropriate threshold levels and control intensity to maintain the number of pests at or below the economic threshold. The modelling and control outcomes presented here extend the results for the system with impulsive interventions at fixed moments.

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**LINDA WAHL**, University of Western Ontario

*The lysis-lysogeny decision in variable environments*

Bacteriophage, viruses that infect bacteria, are the most abundant life form on the planet, and are critical to both ecosystem function and human health. After infecting a host cell, temperate bacteriophage have two life-history strategies: they can immediately kill the cell and release progeny viral particles (lysis); alternatively, they can integrate their genetic material in

the host genome (lysogeny), persisting as an integral part of the host for many generations. Thus, in lysogeny, the predator can choose to become genetically part of the prey. Similar to a bet-hedging strategy, recent experimental evidence suggests that lysogeny may be favoured in variable environments. We examine the evolutionary stability of the lysis-lysogeny decision by analysing invasion reproductive numbers in this intriguing predator-prey system, evolving in the context of a periodic environment.

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**HAO WANG**, University of Alberta

*Multi-scale and qualitative analysis of a stoichiometric algae model in the absence or presence of bacteria*

Algal blooms are becoming a global concern due to the increasing prevalence of eutrophication. Often, algae and bacteria interact, within the well-mixed epilimnion, in a loose commensalistic way. Here we analyze stoichiometric models for algal dynamics and for bacteria-algae dynamics. The algae-only dynamics exhibit rich transient behavior, and the driving biological mechanisms are studied and understood via a multiple time-scale analysis. We further perform global qualitative analysis of both models. There are three dynamical scenarios determined by the basic reproductive numbers of algae and bacteria. We use these models to make specific predictions about how the relative balance of algae and bacteria should change in response to varied nutrient and light availability. The bacteria-algae model successfully reproduces empirical respiration data.

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**LIN WANG**, University of New Brunswick

*Multitype bistability and long transients in a delayed budworm model*

For a delayed budworm model, we show that the delay induce four types of bistability: (i) a node-node bistability (one stable positive equilibrium coexists with another stable positive equilibrium); (ii) a node-cycle bistability (one stable equilibrium coexists with a stable periodic solution); (iii) a cycle-cycle bistability (one stable periodic solution coexists with another stable periodic solution); and (iv) a node-chaotic attractor bistability (one stable positive equilibrium coexists with a chaotic attractor). Moreover, we also show that delay can induce long transients.

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**SHIROU WANG**, University of Alberta

*(De)synchronization for Markov random networks arising from Markov perturbations*

A physical network is naturally subject to noise influences from both external (extrinsic) and internal (intrinsic) sources. The extrinsic noises are usually environmentally related, while the intrinsic ones are typically due to internal uncertainties. A discrete-time, discrete-state (dtds) network with only extrinsic noises is commonly modeled by a discrete random dynamical system (RDS), but the one with only intrinsic noises is often modeled by a Markov chain. In this talk, we will consider a dtds network with both extrinsic and intrinsic noises under the framework of the so-called Markov random network (MRN). In particular, we will discuss the phenomenon and mechanism of (de)synchronizations for MRNs which arise as Markov-perturbations of a synchronized discrete RDS. Characterization of (de)synchronizations will be given from the view points of both probability distributions and dynamical systems.

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**JAMES WATMOUGH**, University of New Brunswick

*A general settlement distribution for dispersal across patchy landscapes*

Many species have a life stage that disperses larger distances than its other stages. For many marine organisms this is a pelagic larval stage. For many familiar insect species, it is an adult stage. Dispersal is across landscapes that are often patchy, with distinct interfaces between suitable and unsuitable habitat for the organism. Some of the central questions in mathematical ecology concern on the effect of patch spacing on persistence and spread of the species. In many analyses, dispersal is modelled by an integral kernel describing the distribution of individuals that survive the dispersal stage. These kernels are complicated by the fact that while dispersing across patchy landscapes, individuals are subject to differential mortality and settlement probabilities in different habitats. These differential rates introduce asymmetries into the kernel. In addition to differential rates across the landscape, individuals may change their behaviour at the patch interfaces. This interface bias may increase

or decrease retention within a patch. We extend previous formulations of dispersal kernels with bias at patch boundaries and derive a general multi-exponential dispersal kernel for a patchy landscape with biased movement across patch interfaces. The kernel is piecewise exponential, with coefficients that can be determined by a straightforward matrix product of the patch parameters. We speculate this matrix formulation of the kernel will simplify numerical computations of integro-difference equation models for population dynamics on patchy landscapes.

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**GAIL S. K. WOLKOWICZ**, McMaster University  
*Analysis of a Simplified Model of Anaerobic Digestion*

Anaerobic digestion is a complex naturally occurring process used for waste and wastewater treatment to produce biogas as a renewable source of energy. The detailed Anaerobic Digestion Model No. 1 (ADM1) includes 32 state variables and is not mathematically tractable. Bornhoft, Hanke-Rauschenbach, and Sundmacher [Nonlinear Dyn., 73 (2013)] introduced a qualitative simplification of this model. The global dynamics of this simplified model will be obtained by first analyzing the limiting system, a model of single species growth in the chemostat in which the response function is non-monotone and the species decay rate is included to complete the theory for the chemostat in this case. Using a Lyapunov function argument and the theory of asymptotically autonomous systems, it will be shown that even in the parameter regime where there is bistability, no periodic orbits exist and every solution converges to one of the equilibrium points. Then two algorithms for stochastically perturbing the parameters of the model will be described. Simulations done with these two algorithms will be compared with simulations done using the Gillespie and tau-leaping algorithms. Finally the dynamics will be compared with the dynamics predicted by the ADM1 model and other simplifications of the ADM1 model.

This is joint work with Tyler Meadows and Marion Weederemann.

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**YINGFEI YI**, University of Alberta  
*A Mesoscopic Ergodic Theorem*

The talk will discuss ergodic properties of mesoscopic systems described by stochastic ordinary differential equations. An ergodic theorem will be presented for systems with rough coefficients which do not necessarily generate flows or semiflows. Applications to mechanical and biological systems will be discussed.

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**PEI YU**, Western University  
*Study of Slow-Fast Motions Using Dynamical Systems Approach*

In this talk, we present a method to analyze certain slow-fast motions in dynamical systems. For singular perturbed dynamical systems, the well-known Geometric Singular Perturbation Method (GSPM) is usually applied to find the special limit cycles - slow-fast periodic solutions. However, many practical problems might be not able or very difficult to be put in the form of singular perturbed equations, but they still exhibit slow-fast motions. For such cases, based on dynamical system theory, we developed a method to identify and analyze slow-fast motions. We will use several biological examples to illustrate our method, and give a comparison between the GSPM and our method.

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**ZHITAO ZHANG**, Academy of Mathematics and Systems Science, the Chinese Academy of Sciences  
*Hénon-Lane-Emden conjecture and related Schrödinger systems*

We have proved Hénon-Lane-Emden conjecture is true for space dimension  $N = 3$  by scaling invariant of the solutions and Sobolev embedding on  $S^{N-1}$ . Then we obtained new Liouville-type theorems and showed Hénon-Lane-Emden conjecture for polyharmonic system holds in a new region, and also proved the generalized Hénon-Lane-Emden conjecture in  $\mathbb{R}^2$  and  $\mathbb{R}^3$ . Moreover, we prove some new results for existence of solutions to related Schrödinger equations and systems.

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**HUAIPING ZHU**, York University  
*Local and global dynamics of models for mosquito-borne diseases*

Mosquitoes and mosquito-borne diseases (MBDs) have become a severe burden of public health. Among many concerns, public health concerns most about the triggers of an outbreak and conditions of repeated infestations. In this talk, I will present some of our dynamical studies to answer the two questions mathematically. I will introduce the local dynamics including local stability and lower codimension bifurcations and their association with the triggering conditions of an outbreak. Besides, I will mainly focus on explaining the global stability, existence and non-existence of periodic solutions, and one related open problem for MBDs.

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**XINGFU ZOU**, University of Western Ontario

*On some R-D models for the sterile insect release method*

Sterile insect release method (SIRM) is a biological control method for troublesome insects. As the fast advancement in genetic techniques and other biological techniques, its range of applications has been greatly enlarged. In this talk, I will begin with a basic ordinary differential equation model for SIRM, and then, move forward to a some reaction diffusion models on a bounded domain. Two releasing strategies for the sterile strain will be discussed, one is domain-wise release and the other is release only on the boundary. By analyzing the models, we obtain some conditions that ensure the success of the sterile release method in eradicating the insects.