
MIKHAIL KARPUKHIN, University of California, Irvine
Isoperimetric inequalities for Laplace eigenvalues.

Eigenvalues of the Laplace operator of Euclidean domains govern many physical phenomena, including heat flow and sound propagation. For this reason inequalities for Laplace eigenvalues have fascinated mathematicians since XIXth century. The following question was first formulated by Lord Rayleigh in his "Theory of sound": which planar domain of given area has the lowest first Dirichlet eigenvalue? This question is the first example of an isoperimetric eigenvalue problem. In the present talk we focus on a more general isoperimetric problem by considering closed surfaces equipped with a Riemannian metric. It turns out that isoperimetric eigenvalue inequalities in this context exhibit a surprising connection to a fundamental geometric object, minimal surface. We will survey some recent advances in the field, including the optimal isoperimetric inequalities for all Laplace eigenvalues on the sphere and the projective plane.