Combinatorial Algebraic Geometry
Géométrie algébrique combinatoire

(Org: **Jenna Rajchgot** (Saskatchewan) and/et **Matthew Satriano** (Waterloo))

AJNEET DHILLON, University of Western Ontario
Root stacks and K-theory

There is a parabolic-orbifold correspondence between vector bundles a root stack and vector bundles with a parabolic structure. It can be used to describe the algebraic K-theory of the root stack. In certain instances the root stack is in fact a quotient stack. Combining these results we obtain a top-up description of equivariant algebraic K-theory.

BALAZS ELEK, University of Toronto
Quiver variety components and minuscule combinatorics

Kashiwara crystals are combinatorial gadgets associated to a representation of a reductive algebraic group that let us understand the structure of the representation in purely combinatorial terms. We will discuss a connection between a geometric and a combinatorial model for the crystals of certain representations, interpreting the lowering operators as choosing a generic subspace in a certain module in the preprojective algebra. This is work in progress joint with Anne Dranowski, Joel Kamnitzer and Calder Morton-Ferguson.

LAURA ESCOBAR, Washington University in St. Louis
Gröbner bases for symmetric determinantal ideals

We give Gröbner bases for a class of combinatorially-defined polynomial ideals which are generated by minors of a generic symmetric matrix. Included within this class are the symmetric determinantal ideals. Each ideal in our class encodes the coordinates and equations for neighborhoods of certain type C Schubert varieties at torus fixed points. This is joint work with Alex Fink, Jenna Rajchgot and Alex Woo.

MEGUMI HARADA, MCMASTER UNIVERSITY
Filtrations and bases of the cohomology rings of regular nilpotent Hessenberg varieties

Hessenberg varieties are a class of subvarieties of the flag variety which are of interest in algebraic and symplectic geometry, combinatorics, and representation theory, among others. Their cohomology rings have been actively studied in recent years due in part to their connection with the well-known Stanley-Stembridge conjecture in combinatorics. I will very briefly recall some of this historical context, and then describe a filtration of the cohomology ring of regular nilpotent Hessenberg varieties. As corollaries, we obtain a natural monomial basis for this cohomology ring, and we also derive, in principle, all linear relations among the (images of the) Schubert classes. This is joint work with T. Horiguchi, S. Murai, M. Precup, and J. Tymoczko.

COLIN INGALLS, Carleton University
McKay graphs of pseudo reflection groups

Pseudo reflection groups are classified by Shepherd and Todd. Up to finitely many exceptions they are the groups G(m,n,p) or the Symmetric groups. We describe the McKay graphs of these groups. This is joint work with R. Buchweitz, E. Faber and M. Lewis.

KELLY JABBUSCH, University of Michigan - Dearborn
Toric surface codes of small dimension

Toric codes are a class of error-correcting codes introduced by Hansen, where a code C is a k-dimensional subspace of $\mathbb{F}_q^n$, coming from a lattice polytope defining a toric variety. In particular, a toric surface code of dimension k is generated by some lattice convex polytope $P \subset \mathbb{R}^2$, where k is the number of lattice points in P. In this talk I'll discuss what is known about toric surface codes of small dimension $(k = 4, 5, 6)$, and how one uses algebraic geometric techniques to analyze such a code. Building on previous work of Soprunov and Soprunova as well as Luo, Yau, Zhang, and Zuo, we'll extend the classification of toric surface codes to dimension $k = 7$. This is joint work with Emily Cairncross, Eli Garcia and Stephanie Ford.

PATRICIA KLEIN, University of Minnesota
Gröbner bases and the Cohen-Macaulay property of Li's double determinantal varieties

We consider double determinantal varieties, a special case of Nakajima quiver varieties. Li Li conjectured that double determinantal varieties are normal irreducible Cohen-Macaulay varieties whose defining ideals have a Gröbner basis given by their natural generators. We use liaison theory to prove this conjecture.

YOAV LEN, Georgia Institute of Technology
Brill–Noether theory of Prym varieties

The talk will revolve around combinatorial aspects of Abelian varieties. I will focus on Pryms, a class of Abelian varieties that occurs in the presence of double covers, and have deep connections with torsion points of Jacobians, bi-tangent lines of curves, and spin structures. I will explain how problems concerning Pryms may be reduced, via tropical geometry, to problems on metric graphs. As a consequence, we obtain new results concerning the geometry of special algebraic curves, and bounds on dimensions of certain Brill–Noether loci.

BRETT NASSERDEN, University of Waterloo
Dynamics in combinatorial geometry

Associated to a surjective endomorphism of smooth projective varieties $f\colon X \to X$ is a number $\lambda_1(f) \geq 1$ which measures the geometric complexity of the dynamics of the morphism. Kawaguchi and Silverman conjectured that this geometric notion agrees with an associated arithmetic notion of dynamical complexity. We will explore some recent work on this conjecture in the context of combinatorial geometry.

KEVIN PURBHOO, University of Waterloo
A new proof of the Shapiro-Shapiro conjecture

In the mid-1990's Boris and Michael Shapiro formulated a remarkable conjecture about real solutions to Schubert problems. It was finally settled in 2005 by Mukhin, Tarasov and Varchenko using machinery from quantum integrable systems. Since then, many applications and generalizations have been found; however, because the MTV proof is fundamentally non-geometric, most of these treat the theorem as a black box. I will talk about a new result, which is simultaneously a generalization of the Shaprio-Shapiro conjecture, and naturally lends itself to a geometric proof. This is joint work with Jake Levinson.

COLLEEN ROBICHAUX, University of Illinois at Urbana-Champaign
Top degree of symmetric Grothendieck polynomials and Castelnuovo-Mumford regularity

We give an easily computable formula for the top degree of the Grothendieck polynomial of a Grassmannian permutation using work of Lenart, and a closely related easily computable formula for the Castelnuovo-Mumford regularity of coordinate rings of affine open patches of Grassmannian Schubert varieties. We connect the latter formula to a conjecture of Kummini-Lakshmibai-Sastry-Seshadri.

SERGIO DA SILVA, University of Manitoba
Understanding the Blow-up of a Subword Complex Along its Boundary

A subword complex is a simplicial complex that describes the structure of the set of reduced subwords of a Weyl group element. They appear in the problem of finding a Gorensteinization for Schubert varieties, where it is known that the blow-up of a Schubert variety along its boundary divisor is Gorenstein. This can be shown by degenerating a Kazhdan-Lusztig variety (which provides local equations for the Schubert variety) to a toric scheme defined by the Stanley-Reisner ideal of a certain subword complex. The blow-up of this Stanley-Reisner scheme along its boundary can be used to prove results about the Kazhdan-Lusztig variety. I will provide a combinatorial description for the blow-up of a subword complex, which characterizes the exceptional components in terms of specific reduced words. I will then apply this characterization to find a combinatorial criteria for determining if a given Kazhdan-Lusztig variety is Gorenstein.

GREGORY G. SMITH, Queen's University
Smooth Hilbert schemes

Hilbert schemes are the prototypical parameter spaces in algebraic geometry—their points correspond to the closed subschemes in $\mathbb{P}^n$ with a fixed Hilbert polynomial. We present numerical conditions on the polynomial that completely characterize when the associated Hilbert scheme is smooth. In the smooth situation, our explicit description of the subschemes being parametrized also provides new insights into the global geometry of the Hilbert scheme. This talk is based on joint work with Roy Skjelnes (KTH).