Algebraic Geometry and Representation Theory
Géométrie algébrique et théorie de la représentation

(Org: **Lisa Jeffrey** (Toronto) and/et **Steven Rayan** (Saskatchewan))

PETER CROOKS, Northeastern University
The cohomology rings of regular Hessenberg varieties

Hessenberg varieties form a distinguished class of subvarieties in the flag variety, and their study is central to themes at the interface of combinatorics, geometric representation theory, and symplectic geometry. Such themes include the Stanley–Stembridge and Shareshian–Wachs conjectures, in which the cohomology rings of Hessenberg varieties feature prominently.

I will provide an invariant-theoretic description of the cohomology rings of regular Hessenberg varieties, emphasizing the roles played by Tymoczko's dot action, the Grothendieck–Springer resolution, Deligne's local invariant cycle theorem, and topological monodromy. Our results build upon those of Brosnan–Chow, Abe–Harada–Horiguchi–Masuda, and Abe–Horiguchi–Masuda–Murai–Sato.

This represents joint work with Ana Balibanu.

AJNEET DHILLON, University of Western Ontario
Essential Dimension of parabolic bundles.

Essential dimension is a kind generic of parameter count for a family. In this talk we will discuss the essential dimension of the moduli stack of vector bundles with parabolic structure on a smooth projective curve. This is joint work with Dinesh Valluri. It extends prior work with Indranil Biswas and Norbert Hoffmann.

JACK DING, University of Toronto
The Atiyah-Bott Localization Formula for $\Omega SU(2)$

The Atiyah-Bott fixed point theorem for elliptic complexes is a powerful tool to compute the Lefschetz number of an elliptic operator in terms of data around the fixed points of a compact Lie group action. It has various applications in geometry and representation theory, such as the Weyl character formula for semisimple Lie algebras. In that case, the rational functions one multiplies at each fixed point is given by the Weyl denominator.

The based loop group of a compact group K has a natural action by its maximal torus T and a rotation action by the circle S^1, these two actions commute. We extend the Atiyah-Bott formula to the based loop group of $\Omega SU(2)$ and provide a formula for the rational functions one must multiply at each fixed point of the $T \times S^1$ action. This is done by applying the Atiyah-Bott theorem on a filtration of $\Omega SU(2)$ comprised of finite-dimensional spaces and taking a limit.

ELISHEVA ADINA GAMSE, University of Toronto Mississauga
Vanishing theorems in the cohomology ring of the moduli space of parabolic vector bundles over a Riemann surface

Let Σ be a compact connected oriented 2-manifold of genus $g \geq 2$, and let p be a point on Σ. We define a space $S_g(t)$ consisting of certain irreducible representations of the fundamental group of $\Sigma \setminus p$, modulo conjugation by $SU(n)$. This space has interpretations in algebraic geometry, gauge theory and topological quantum field theory; in particular if Σ has a Kahler structure then $S_g(t)$ is the moduli space of parabolic vector bundles of rank n over Σ. For $n = 2$, Weitsman considered a tautological line bundle on $S_g(t)$, and proved that the $2g^{th}$ power of its first Chern class vanishes, as conjectured by Newstead. In this talk I will present his proof and outline my extension of his work to $SU(n)$ and to $SO(2n+1)$. I will also explore the case where Σ has multiple marked points.

MICHAEL GROECHENIG, University of Toronto
Rigid representations of fundamental groups

In the 90s Simpson conjectured an integrality property for the monodromy of rigid representations of fundamental groups of smooth projective varieties. In my talk I will explain how to prove this for cohomologically rigid representations. This is joint work with Hélène Esnault.

IVA HALACHEVA, Northeastern University
Branching in Schubert calculus

Usual Schubert calculus problems concern multiplication, i.e. the pullback in cohomology along inclusion of a diagonal. Another natural map to study is the pullback of the inclusion of Sp(2n) into GL(2n), in equivariant cohomology of the corresponding Grassmannians. I will present a puzzle rule which describes where this map takes Schubert classes, as well as an extension to cotangent bundles using Lagrangian correspondences between symplectic resolutions, and Maulik-Okounkov classes. This is joint work with Allen Knutson and Paul Zinn-Justin.

JACQUES HURTUBISE, McGill University
Connections and moduli of bundles.

This talk concerns the symplectic geometric and differential geometric aspects of the moduli space of connections on a compact Riemann surface X. Fix a theta characteristic $K_X^{1/2}$ on X; it defines a theta divisor on the moduli space $\mathcal{M}$ of stable vector bundles on X of rank r degree zero. Given a vector bundle $E \in \mathcal{M}$ lying outside the theta divisor, we construct a natural holomorphic connection on E that depends holomorphically on E. Using this holomorphic connection, there is a canonical holomorphic isomorphism between:

1. the moduli space $\mathcal{C}$ of pairs (E, D), where $E \in \mathcal{M}$ and D is a holomorphic connection on E, and
2. the space $\mathrm{Conn}(\Theta)$ given by the sheaf of holomorphic connections on the line bundle on $\mathcal{M}$ associated to the theta divisor.

The above isomorphism between $\mathcal{C}$ and $\mathrm{Conn}(\Theta)$ is symplectic structure preserving, and it moves holomorphically as X runs over a holomorphic family of Riemann surfaces. (joint with I. Biswas)

JOEL KAMNITZER, University of Toronto
BFN Springer theory

Given a representation of a reductive group, Braverman-Finkelberg-Nakajima have defined a remarkable Poisson variety called the Coulomb branch. Their construction of this space was motivated by considerations from supersymmetric gauge theories and symplectic duality. The coordinate ring of this Coulomb branch is defined as a kind of cohomological Hall algebra; thus it makes sense to develop a type of "Springer theory" to define modules over this algebra. In this talk, we will explain this BFN Springer theory and give many examples. In the toric case, we will see a beautiful combinatorics of polytopes. In the quiver case, we will see connections to the representations of quivers over power series rings. In the general case, we will explore the relation between these Springer fibres and quasimap spaces.

MAXENCE MAYRAND, University of Toronto
Kempf-Ness type theorems and Nahm's equations

From one point of view, Kempf-Ness type theorems give explicit complex-algebraic descriptions of some real symplectic quotients. This is particularly interesting and not obvious when the symplectic form is transcendental and the manifold is non-compact, as is often the case when studying certain gauge-theoretical moduli spaces. In this talk, I will present a general result in this context (for complex affine varieties with non-standard Kähler structures and shifted moment maps) and explain how Nahm's equations, a system of ODEs extracted from the self-dual Yang-Mills equations, provide non-trivial examples.

RUXANDRA MORARU, University of Waterloo

The Vafa-Witten equations and T-branes

The Vafa-Witten equations are a higher-dimensional analogue of the Hitchin equations on compact Riemann surfaces for oriented four-manifolds. On a compact complex surface, their solutions are polystable Higgs bundles (with Higgs fields taking values in a vector bundle twisted by the canonical bundle of the surface); T-branes are solutions whose Higgs fields are non-abelian. In this talk, we describe the geometry of T-branes and prove, in particular, that they can only exist on properly elliptic surfaces and surfaces of general type. We also give examples. This is work in progress with Fernando Marchesano and Raffaele Savelli.

MATEJ PENCIAK, Northeastern University

A spectral description of the spin Ruijsenaars-Schneider system.

The Ruijsenaars-Schneider system is a many-particle integrable system which can be viewed as a relativistic analogue of the better known Calogero-Moser system. In this talk I will provide a moduli-theoretic description of the phase space for the RS system in terms of spectral sheaves living in particular $\mathbb{P}^1$ bundles over elliptic curves. In this description the flows are the naturally associated Hitchin flows. This yields an interpretation of the Lax matrix for the RS system as a Higgs field for the associated Hitchin system.

BRENT PYM, McGill University

Period domains for quantum planes

A celebrated theorem of Artin, Tate and Van den Bergh explains that noncommutative analogues of the projective plane (in the form of 3d Artin-Schelter regular algebras) are classified up to isomorphism by automorphisms of cubic curves. I will outline an alternative viewpoint on this classifying data, based on Katzarkov-Kontsevich-Pantev's theory of noncommutative Hodge structures: it is the shadow of a canonical mixed Hodge structure on the topological K-theory of an associated differential graded category. As an application, we realize a proposal of Kontsevich, giving the first calculation of his deformation quantization formula in the case of rational Poisson surfaces, recovering Feigin and Odesskii's explicit presentations of noncommutative algebras in terms of elliptic theta functions. This talk is based on joint work with Theo Raedschedlers and Sue Sierra.

ALEX WEEKES, University of British Columbia

Deformations of affine Grassmannian slices

Affine Grassmannian slices are interesting singular affine algebraic varieties: by the geometric Satake equivalence, their singularities are closely related to the representation theory of reductive groups. These varieties also arise as Coulomb branches for $3d\ \mathcal{N}=4$ theories by the recent work of Braverman-Finkelberg-Nakajima. They have Poisson structures, and are examples of conical symplectic singularities. Conical symplectic singularities have a nice deformation theory, by work of Namikawa, Losev and others. In the case of affine Grassmannian slices, I will describe how this deformation theory is related to the Beilinson-Drinfeld Grassmannian. This is work in progress with Gwyn Bellamy, Dinakar Muthiah and Oded Yacobi.

JONATHAN WEITSMAN, Northeastern University

Enhanced symmetry in the semiclassical category and characters of loop groups

Enhanced symmetry in the semiclassical category and characters of loop groups.

We consider a version of Weinstein's symplectic category, adapted for the case of quasi-Hamiltonian G-spaces. We show that semiclassical quantization in this setting produces the Kac character formula, in analogy with the construction of the Weyl character formula by Guillemin and Sternberg in the symplectic setting. We show that this construction gives a natural action of the modular group on the Kac characters, which we conjecture agrees with Kac's SL(2,Z) action. We conjecture also that a similar construction should give rise to enhanced symmetries—-that is, symmetries of the quantization that do not arise from symmetries of the underlying classical system—also in some other examples.