ALEXEY F. SHEVYAKOV, University of Saskatchewan
Symbolic computation of symmetries and first integrals in dynamical systems

While the solution structure of dynamical systems given by ordinary differential equations (ODE) and ODE systems is well understood, and the solution behaviour can be analyzed locally and globally using various analytical methods, for the vast majority of nonlinear systems, it is not possible to write down the *general solution* in a closed form. In many cases, however, it is possible to reduce the differential order of the ODEs. Order can be reduced when one knows a conserved quantity (a *first integral*) that is constant on solutions, and/or a local *symmetry group* that leaves the ODE system invariant. A sufficient number of first integrals and/or local symmetries known for a given ODE model leads to its complete integration.

Lie groups of point, contact, and higher-order symmetries can be systematically computed using the Lie's algorithm, which, however, may require heavy computations.

Similarly, conservation laws of partial differential equations, and as a special case, first integrals of ODEs, can be computed using the direct (multiplier) method and the Euler differential operator that annihilates divergences. Such computations are also algorithmic, but for nontrivial examples, can be computationally demanding.

In this talk, we will demonstrate symbolic computations of symmetries and first integrals of ODEs using the `Maple`-based `GeM` symbolic software package. Examples of dynamical systems based on ODEs and nonlinear PDEs, such as shallow water models and the b-family of peakon equations, will be presented.