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Persistent homotopy theory

Suppose that $X \subset Y$ are data sets in a metric space Z , and suppose that $r > 0$. A theorem of Blumberg-Lesnick asserts that if $y \in Y$ satisfies $d(x, y) < r$ for some $x \in X$, then the inclusion of systems $V(X) \subset V(Y)$ has homotopy interleaving distance $< 2r$. This result can be proved with an order complex argument.

A bounded distance criterion for the inclusion $X_{dis}^{k+1} \subset Y_{dis}^{k+1}$ of subsets of $k + 1$ distinct points for X and Y implies that the inclusion of systems $L_{*,j}(X) \subset L_{*,j}(Y)$ has a homotopy interleaving distance $< 2r$, for $j \leq k$.

The space $D(Z)$ of finite subsets of a metric space Z is the platform for the Blumberg-Lesnick stability theorem. Homotopy interleavings for inclusions of systems $V(X) \subset V(Y)$ specify the local behaviour of these systems for $X \subset Y$, as Y approaches X in the Hausdorff metric on $D(Z)$.