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The Regularity Radius of Delone Sets

Delone sets are uniformly discrete point sets in Euclidean d-space used in the modeling of crystals. The mathematical theory describing the relationship between the local and global structure of a crystal was initiated by Delone, Dolbilin, Shtogrin, and Galiulin in the 1970's, and resulted in the Local Theorem for Delone Sets. Its main goal is to understand in mathematical terms how congruence of local atomic arrangements forces global structural regularity or periodicity. There are two types of Delone sets that account for regularity or periodicity, respectively, namely the regular systems (orbits of a single point under a crystallographic group) or multiregular systems (unions of finitely many point orbits under a crystallographic group). The local theory searches for local conditions on a Delone set X that guarantees the emergence of a crystallographic group of symmetries producing X as an orbit set consisting of a single point orbit or finitely many point orbits, respectively. There are interesting new results for the regularity radius of Delone sets. The regularity radius is the smallest number such that each Delone set X in d-space with mutually congruent point clusters (point neighborhoods) of this radius is a regular system. We discuss old and new results from the local theory of Delone sets, including a lower bound for the d-dimensional regularity radius which is linear in R, the radius of the largest empty ball of X, as well as an upper bound in dimension 3. This is joint work with a group of mathematicians and crystallographers.