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Bohr-Sommerfeld-Heisenberg quantization of the mathematical pendulum

For a completely integrable Hamiltonian system, Bohr-Sommerfeld quantization of the action variables gives rise to a space of quantum states and a complete set of commuting observables acting of this space of states. Bohr-Sommerfeld theory does not provide operators of transition between the eigenstates of operators corresponding to the actions. These transitions are accounted for by shifting operators. Their existence for the mathematical pendulum will be discussed in the talk. The commutation relations satisfied by the shifting operators are the same as the commutation relations satisfied by formal quantization of the exponential function of i times theta, where theta is an angle in the action angle coordinates for the integrable system. If theta were a single-valued function, then its Hamiltonian vector field would generate a local group of local symplectomorphisms of the phase space preserving the Bohr-Sommerfeld polarization by level sets of the Hamiltonian. This flow would lift to a local group of local quantomorphisms of the quantum line bundle. Since the angle theta is a multi-valued function, this local group of quantomorphisms is not well defined. However, the shifting operators, given by evaluating the lifted flow of quantomorphisms at $t = h$ is well defined and corresponds to the operators of multiplication by the exponential function of i times theta. The existence of shifting operators answers Heisenberg's criticism of Bohr-Sommerfeld theory.