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Entropic convexity and the Einstein equation for gravity

On a Riemannian manifold, lower Ricci curvature bounds are known to be characterized by geodesic convexity properties of various entropies with respect to the Kantorovich-Rubinstein-Wasserstein square distance from optimal transportation. These notions also make sense in a (nonsmooth) metric measure setting, where they have found powerful applications. In this talk I describe the development of an analogous theory for lower Ricci curvature bounds in timelike directions on a Lorentzian manifold. In particular, by lifting fractional powers of the Lorentz distance (a.k.a. time separation function) to probability measures on spacetime, I show the strong energy condition of Penrose is equivalent to geodesic concavity of the Boltzmann-Shannon entropy there. Parallel work of Mondino and Suhr gives also the complementary upper bound, hence a reformulation of the Einstein field equations of general relativity in terms of the convexity properties of the entropy.

See preprint at http://www.math.toronto.edu/mccann/papers/GRO.pdf