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Bounded total variation of Kaczmarz trajectories for arbitrary control sequences

The Kaczmarz method is a popular iterative algorithm for solving linear systems of equations where the next iterate is determined by projecting the current iterate onto the solution hyperplane associated with any one of the individual equations. The sequence of the chosen hyperplanes is called the control sequence. For consistent systems, this iteration is known to always converge, and in particular, to a point in the intersection of the hyperplanes that are visited infinitely often. Linear convergence rates can be achieved with control sequences that are sufficiently regular, but for arbitrary control sequences the convergence can also be arbitrarily slow. In this talk (in joint work with Thao Nguyen), we show that regardless of the control sequence, the convergence takes place in a stronger sense in that the trajectories always have bounded variation (this is sometimes called absolute convergence), and their total variation can be bounded uniformly over all control sequences. Our result also holds for a more general version of the Kaczmarz method that incorporates relaxation.