MAURICE POUZET, Claude-Bernard University and University of Calgary
A topological interpretation of de Jongh -Parikh theorem and applications

An ordered set P is partially well-ordered (pwo) if it contains no infinite descending chain and if all its antichains are finite. A famous result of de Jongh and Parikh (1977) asserts that among the linear extensions of P, one has a largest order type, say $\ell(P) = \omega^{\beta_0} \cdot m_0 + \cdots + \omega^{\beta_{k-1}} \cdot m_{k-1}$. We give a topological interpretation of the coefficients of $\ell(P)$ in terms of Cantor-Bendixson rank. The set $\mathbf{Id}(P)$ of ideals (non-empty up-directed initial segments) of P, once equipped with the topology induced by topology on the power set $\mathcal{P}(P)$, being a compact scattered topological space, we define a partition of P into k blocks P_i so that β_i is the rank of $\mathbf{Id}(P_i)$ and m_i the number of elements having that rank. We illustrate our result with the poset P made of words over a finite alphabet A.

We compute the ordinal length of the set $\mathbf{I}_{<\omega}(P)$ of finitely generated ideals of a wqo P that is embeddable into $[\omega]^{<\omega}$, the poset of finite subsets of ω. Building on this result, we compute the ordinal length of the set of monomial ideals in n variables. This answers a question of Aschenbrenner and Pong, 2004.

This is a joint work with C.Delhommé (Université de la Réunion) and M.Sobrani (University of Fes, Morocco).