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Well-posedness of fourth-order Schrödinger equation with derivative nonlinearities

In this talk, we study well-posedness of the Cauchy problem to the forth-order nonlinear Schrödinger equations with $\gamma \in \{1,2,3\}$-times derivative nonlinearities in Sobolev space $H^s(\mathbb{R})$:

$$\begin{cases} i\partial_t u + \partial_x^4 u = G\left((\partial_x^k u)_{k\le\gamma}, (\partial_x^k \bar{u})_{k\le\gamma}\right), & (t,x) \in I \times \mathbb{R}, \\ \quad u|_{t=0} = u_0 \in H^s(\mathbb{R}), \end{cases} \tag{1}$$

where $u : I \times \mathbb{R} \to \mathbb{C}$ is an unknown function, $I := [-T, T]$ denotes the existence time interval of the function u, $u_0 \in H^s(\mathbb{R})$ is a prescribed function, and for $s \in \mathbb{R}$, $H^s(\mathbb{R})$ denotes $L^2(\mathbb{R})$-based Sobolev space. For $m \in \mathbb{N}$ with $m \geq 3$, we mainly consider the m-th order nonlinearity G of the form

$$G(z) = G(z_1, \cdots, z_{2(\gamma+1)}) := \sum_{|\alpha|=m} C_\alpha z^\alpha,$$

where $C_\alpha \in \mathbb{C}$ with $\alpha \in (\mathbb{N} \cup \{0\})^{2(\gamma+1)}$ are constants. The purpose of this talk is to improve the previous results obtained by several Mathematicians, that is, to treat more general nonlinearity and to prove local well-posedness of the problem in lower order Sobolev space $H^s(\mathbb{R})$. Our proof of the well-posedness result is based on the contraction argument on a suitable function space, via the Strichartz estimates, Kato-type smoothing estimates, Kenig-Ruiz estimates, Maximal function estimates, a linear estimate for inhomogeneous term, the bilinear Strichartz type estimate and the Littlewood-Paley theory.