Geometric Analysis and Spectral Geometry
Analyse géométrique et géométrie spectrale
(Org: **Richard Froese** (University of British Columbia) and/et **Alexandre Girouard** (Laval University — Université Laval))

NICOLAU SARQUIS AIEX, University of British Columbia
Index estimate of self-shrinkers.

We show that the Morse index of a self-shrinker is greater or equal to $\frac{2g+r-1}{3}+1$, where r is the number of asymptotically conical ends. This is a generalization of previous results of McGonagle and more recently Impera-Rimoldi-Savo.

ALBERT CHAU, University of British Columbia
Complete solutions to Kahler Ricci flow on quasi-projctive varieties.

ABSTRACT: In this talk I will discuss the existence of complete non-compact solutions to the Kahler Ricci flow on quasi projective varieties M emerging from zero Lelong number currents on the underlying compact manifold. The discussion will include complete solutions to the flow emerging from complete initial metrics with unbounded curvature, and also "instantaneously complete" solutions. Special attention will be given to cusp like metrics and solutions. A Kahler metric g is called cusp-like on M if it is equivalent to the standard hyperbolic punctured disc model in complex directions approaching D. The talk will be based on joint with with Liangming Shen and Ka Fai Li.

JINGYI CHEN, The University of British Columbia
Recent progress on Hamiltonian stationary Lagrangian submanifolds

We shall discuss recent work on regularity, compactness, extensions for Hamiltonian stationary Lagrangian submanifolds in the complex space $\mathbb{C}^n$. The results are based on joint work with Micah Warren.

DONATO CIANCI, University of Michigan
On branched minimal immersions of surfaces by first eigenfunctions

Montiel and Ros proved that for each conformal structure on a compact surface there is at most one metric which admits a minimal immersion into some unit sphere by first eigenfunctions. In this talk I'll discuss a generalization of this theorem to the setting of metrics with conical singularities induced from branched minimal immersions by first eigenfunctions into spheres. In particular, we will see that the properties of such metrics induced from the 2-sphere differ significantly from the properties of those induced from an m-sphere with $m>2$. Our primary motivation is that metrics which maximize the first non-zero Laplace eigenvalue are induced by minimal branched immersions into spheres. With this in mind, I'll also discuss these results in the context of such eigenvalue optimization problems for closed surfaces. This work is joint with Mikhail Karpukhin and Vladimir Medvedev.

ALEXANDRE GIROUARD, Université Laval
Steklov eigenvalues of submanifolds with prescribed boundary in Euclidean space

We obtain upper and lower bounds for Steklov eigenvalues of submanifolds with prescribed boundary in Euclidean space. A general upper bound is proved, which depends only on the geometry of the fixed boundary and on the measure of the interior. Sharp lower bounds are given for hypersurfaces of revolution with connected boundary: we prove that each eigenvalue is uniquely minimized by the ball. We also observe that each surface of revolution with connected boundary is isospectral to the disk. This is joint work with Bruno Colbois and Katie Gittins.

CAROLYN GORDON, Dartmouth College
Transplantation and isogeny of intermediate Jacobians of Kahler manifolds

We give a general method for constructing compact Kähler manifolds X_1 and X_2 whose intermediate Jacobians $J^k(X_1)$ and $J^k(X_2)$ are isogenous for each k, and we exhibit examples. Under an additional hypothesis, the Hodge Laplacians acting on forms of type (p,q) are isospectral for all choices of (p,q). The method is based upon the algebraic transplantation formalism arising from Sunada's technique for constructing pairs of compact Riemannian manifolds whose Laplace spectra are the same.

This is joint work with Eran Makover, Bjoern Muetzel and David Webb.

EVANS HARRELL, Georgia Institute of Technology
Two-term asymptotically sharp bounds for eigenvalue means of the Laplacian

We consider the the eigenvalue spectrum of the Laplacian on a domain and use the averaged variational principle to produce lower-order corrections to the celebrated inequalities of Berezin-Li-Yau in the Dirichlet case and of Kröger in the Neumann case, which are sharp in the high-energy régime. We also produce complementary bounds, i.e., an analogue of the Berezin-Li-Yau inequality for the Neumann problem and an analogue of the Kröger inequalitty for the Dirichlet problem. This is joint work with Joachim Stubbe and Luigi Provenzano.

ASMA HASSANNEZHAD, University of Bristol
On a relation between Dirichlet-to-Neumann and Laplacian spectra on manifolds

The Dirichlet-to-Neumann operator is a first order elliptic pseudodifferential operator. It acts on smooth functions on the boundary of a Riemannian manifold and maps a function to the normal derivative of its harmonic extension. The eigenvalues of the Dirichlet-to-Neumann map are also called Steklov eigenvalues. It has been known that the geometry of the boundary has a strong influence on the Steklov eigenvalues. In this talk, we show that for every $k \in \mathbb{N}$, the k-th Steklov eigenvalue σ_k is comparable to the square root of the k-th eigenvalue of the Laplacian $\sqrt{\lambda_k}$ on the boundary. More precisely, we show that there exists a constant C depending only on geometry near the boundary such that $|\sigma_k - \sqrt{\lambda_k}| < C$. This is joint work with Bruno Colbois and Alexander Girouard.

DIMA JAKOBSON, McGill University
Zero and negative eigenvalues of the conformal laplacian

This is joint work with Y. Canzani, R. Gover, R. Ponge, A. Hassannezhad and M. Levitin. We study conformal invariants that arise from nodal sets and negative eigenvalues of conformally covariant operators, which include the Yamabe and Paneitz operators. We give several applications to curvature prescription problems. We establish a version in conformal geometry of Courant's Nodal Domain Theorem. We prove that the Yamabe operator can have an arbitrarily large number of negative eigenvalues on any manifold of dimension greater than or equal to 3. We show that 0 is generically not an eigenvalue of the conformal Laplacian. If time permits, we shall discuss related results for weighted graphs.

RICHARD LAUGESEN, University of Illinois, Urbana-Champaign
From Neumann to Steklov and beyond, via Robin: the Weinberger way

Weinberger showed the first nontrivial Neumann eigenvalue of the Laplacian is maximal for the ball, among domains of fixed volume. Brock did the same for the first nontrivial Steklov eigenvalue. We connect these two results by generalizing to the second Robin eigenvalue of the Laplacian. The Robin parameter here is negative, lying in the range between the Neumann and Steklov eigenvalues and going even somewhat beyond the Steklov regime.

RENAN ASSIMOS MARTINS, Max Planck Institute for Mathematics in the Sciences
A remark on the Geometry of Some Maximum Principles

A cornerstone in the theory of minimal surfaces is Bernstein's theorem, stating that the only entire minimal graphs in Euclidean 3-space are planes. The effort of many mathematicians lead to several generalizations of this statement. The works of Simons, Bombieri-De Giorgi-Giusti, Moser, Lawson-Osserman and Hildebrandt-Jost-Widman are examples of such results: the first two proving that this theorem is true for minimal hypersurfaces of dimension up to 7 and false for higher dimensions; the third proves the theorem for any minimal hypersurface adding a bounded slope condition; for higher codimensionas, L-O have provided counterexamples even under the extra hypothesis on the slope, while the last work cited gave a stronger condition on the slope to obtain a Bernstein type result. In our work, we present a generalization of Moser's theorem in codimension 2. More precisely, if $f : \mathbb{R}^n \longrightarrow \mathbb{R}^2$, $f(x) = (f^1(x), f^2(x))$ is a smooth map defined everywhere in $\mathbb{R}^n$, $M = (x, f(x))$ is a minimal submanifold in $\mathbb{R}^2$ and there exists a number $\beta_0 < +\infty$ s.t. $\Delta_f \leq \beta_0$ for all $x \in \mathbb{R}^p$, where $\Delta_f(x) := \left\{det\left(\delta_{\alpha\beta} + \sum_i f^i_{x_\alpha}(x) f^i_{x_\beta}(x)\right)\right\}^{\frac{1}{2}}$, then $f^i : \mathbb{R}^n \longrightarrow \mathbb{R}$, $i = 1, 2$ are linear functions on $\mathbb{R}^n$. To prove this theorem we develop general techniques to study the geometry of subsets of a complete Riemannian manifold that contain no image of non-constant harmonic maps. We use this to study regions in a Grassmannian manifold with this property, since the Gauss map of a minimal submanifold is a harmonic map with image into $G^+_{p,n}$. With these ideas we obtain Bernstein type results.

DAVID SHER, DePaul University

The Steklov eigenvalue problem on polygons

Title: The Steklov eigenvalue problem is an eigenvalue problem like the Dirichlet or Neumann problem, but with the eigenvalue parameter appearing in the boundary condition rather than in Laplace's equation. It is known that there are surprisingly sharp eigenvalue asymptotics for the Steklov problem on surfaces with smooth boundary. I will explain what happens to these asymptotics when the surface has corners; the answer involves a fascinating scattering-like phenomenon along the boundary. This talk is based on joint work in progress with M. Levitin, L. Parnovski, and I. Polterovich.

ELISABETH STANHOPE, Lewis & Clark College

Bounding the multiplicity of the Steklov eigenvalues of an orbisurface

The Steklov spectrum of a Riemannian orbifold with boundary is the eigenvalue spectrum of the Dirichlet-to-Neumann operator associated to the orbifold. This operator has applications in electrical impedance tomography, for example. We discuss bounding the multiplicity of the kth Steklov eigenvalue of a 2-orbifold in terms of the genus of the orbifold and the structure of its singular set.

CRAIG SUTTON, Dartmouth