RENAN ASSIMOS MARTINS, Max Planck Institute for Mathematics in the Sciences
A remark on the Geometry of Some Maximum Principles

A cornerstone in the theory of minimal surfaces is Bernstein's theorem, stating that the only entire minimal graphs in Euclidean 3-space are planes. The effort of many mathematicians lead to several generalizations of this statement. The works of Simons, Bombieri-De Giorgi-Giusti, Moser, Lawson-Osserman and Hildebrandt-Jost-Widman are examples of such results: the first two proving that this theorem is true for minimal hypersurfaces of dimension up to 7 and false for higher dimensions; the third proves the theorem for any minimal hypersurface adding a bounded slope condition; for higher codimensionas, L-O have provided counterexamples even under the extra hypothesis on the slope, while the last work cited gave a stronger condition on the slope to obtain a Bernstein type result. In our work, we present a generalization of Moser's theorem in codimension 2. More precisely, if $f : \mathbb{R}^n \longrightarrow \mathbb{R}^2$, $f(x) = (f^1(x), f^2(x))$ is a smooth map defined everywhere in $\mathbb{R}^n$, $M = (x, f(x))$ is a minimal submanifold in $\mathbb{R}^2$ and there exists a number $\beta_0 < +\infty$ s.t. $\Delta_f \leq \beta_0$ for all $x \in \mathbb{R}^p$, where $\Delta_f(x) := \left\{det\left(\delta_{\alpha\beta} + \sum_i f^i_{x_\alpha}(x) f^i_{x_\beta}(x)\right)\right\}^{\frac{1}{2}}$, then $f^i : \mathbb{R}^n \longrightarrow \mathbb{R}$, $i = 1, 2$ are linear functions on $\mathbb{R}^n$. To prove this theorem we develop general techniques to study the geometry of subsets of a complete Riemannian manifold that contain no image of non-constant harmonic maps. We use this to study regions in a Grassmannian manifold with this property, since the Gauss map of a minimal submanifold is a harmonic map with image into $G^+_{p,n}$. With these ideas we obtain Bernstein type results.