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**Distributions in Analytic Number Theory**  
**Distributions dans la théorie analytique des nombres**

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**KIRSTI BIGGS**, University of Bristol  
*Efficient congruencing in ellipsephic sets*

An ellipsephic set is a subset of the natural numbers whose elements have digital restrictions in some fixed base. We bound the number of solutions to a Vinogradov system of equations in which our variables are drawn from certain sparse ellipsephic sets—a key example is those integers whose digits in a given base are squares—using a version of Wooley’s efficient congruencing method. In this talk, I will outline the key ideas from the proof and discuss potential applications and generalisations.

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**CHANTAL DAVID**, Concordia University  
*Averages of cubic Gauss sums over functions fields*

We present in this talk some explicit results for averages of cubic Gauss sums over functions fields. Contrary to the quadratic Gauss sums, the behavior of the cubic Gauss sums is chaotic, and in order to address the distribution of cubic Gauss sums, one must use the deep work of Kubota, Heath-Brown and Patterson on automorphic forms for the metaplectic group. This was generalized to the context of function fields by Hoffstein and Patterson. By evaluating precisely the average of those cubic Gauss sums, one gets some unexpected cancellation between the dual and the main error term of the moments of cubic Dirichlet L-functions over number fields.

This is joint work with M. Lalin and Alexandra Florea.

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**LUCILE DEVIN**, UOttawa  
*Chebyshev’s bias for products of irreducible polynomials – II*

(joint with X. Meng) For any finite field  $\mathbb{F}$ , and for any positive integer  $k$ , we obtain an asymptotic for the difference of functions counting products of  $k$  irreducible polynomials with coefficients in  $\mathbb{F}$  among different arithmetic progressions. We unconditionally derive the existence of the limiting distribution of this difference. In contrast to the “translation” in the ring of integers, we show the existence of complete biases in the function field setting, that is the difference function may have constant sign.

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**DANIEL FIORILLI**, University of Ottawa / CNRS  
*Large values of the variance of primes in arithmetic progressions*

This is joint work with Greg Martin. Hooley conjectured an asymptotic for the variance of primes in arithmetic progressions. We will show that in a certain range, this variance is significantly larger than expected.

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**AYLA GAFNI**, University of Rochester  
*Extremal primes for elliptic curves without complex multiplication*

Fix an elliptic curve  $E$  over  $\mathbb{Q}$ . An “extremal prime” for  $E$  is a prime  $p$  of good reduction such that the number of rational points on  $E$  modulo  $p$  is maximal or minimal in relation to the Hasse bound. In this talk, I will discuss what is known and conjectured about the number of extremal primes  $p \leq X$ , and give the first non-trivial upper bound for the number of such primes when  $E$  is a curve without complex multiplication. The result is conditional on the hypothesis that all the symmetric power  $L$ -functions associated to  $E$  are automorphic and satisfy the Generalized Riemann Hypothesis. In order to obtain this

bound, we use explicit equidistribution for the Sato-Tate measure as in recent work of Rouse and Thorner, and refine certain intermediate estimates taking advantage of the fact that extremal primes have a very small Sato-Tate measure.

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**JACK KLYS**, University of Calgary

*Moments of unramified 2-group extensions of quadratic fields*

We discuss the Cohen-Lenstra heuristics from the point of view of counting unramified number field extensions of quadratic fields. We will focus on the specific case of 2-group extensions of quadratic fields which has proven to be more tractable in recent years. We will put forth a conjecture about asymptotics and distributions of such extensions (beyond those considered by Cohen and Lenstra) and discuss our recent progress on this in the case of extensions with Galois groups which are central extensions of  $\mathbb{F}_2^n$  by  $\mathbb{F}_2$ .

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**MATILDE LALIN**, Université de Montréal

*The mean value of cubic L-functions over function fields*

We present results about the first moment of  $L$ -functions associated to cubic characters over  $\mathbb{F}_q(T)$  when  $q \equiv 1 \pmod{3}$ . The case of number fields was considered in previous work, but never for the full family of cubic twists over a field containing the third roots of unity. We will explain how to obtain an asymptotic formula with a main term, which relies on using results from the theory of metaplectic Eisenstein series about cancellation in averages of cubic Gauss sums over function fields. We will also discuss the case  $q \equiv 2 \pmod{3}$ .

This is joint work with C. David and A. Florea.

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**ALLYSA LUMLEY**, York University

*Distribution of Values of L-functions associated to Hyperelliptic Curves over Finite Fields*

In 1992, Hoffstein and Rosen proved a function field analogue to Gauß' conjecture (proven by Siegel) regarding the class number,  $h_D$ , of a discriminant  $D$  by averaging over all polynomials with a fixed degree. In this case  $h_D = |\text{Pic}(\mathcal{O}_D)|$ , where  $\text{Pic}(\mathcal{O}_D)$  is the Picard group of  $\mathcal{O}_D$ . Andrade later considered the average value of  $h_D$ , where  $D$  is monic, squarefree and its degree  $2g + 1$  varies. He achieved these results by calculating the first moment of  $L(1, \chi_D)$  in combination with Artin's formula relating  $L(1, \chi_D)$  and  $h_D$ . Later, Jung averaged  $L(1, \chi_D)$  over monic, squarefree polynomials with degree  $2g + 2$  varying. Making use of the second case of Artin's formula he gives results about  $h_D R_D$ , where  $R_D$  is the regulator of  $\mathcal{O}_D$ .

For this talk we discuss the complex moments of  $L(1, \chi_D)$ , with  $D$  monic, squarefree and degree  $n$  varying. Using this information we can describe the distribution of values of  $L(1, \chi_D)$  and after specializing to  $n = 2g + 1$  we give results about  $h_D$  and specializing to  $n = 2g + 2$  we give results about  $h_D R_D$ .

If time permits, we will discuss similar results for  $L(\sigma, \chi_D)$  with  $\frac{1}{2} < \sigma < 1$ .

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**AMITA MALIK**, Rutgers University

*Distribution of zeros of derivatives of the Riemann  $\xi$ -function*

For the completed Riemann zeta function  $\xi(s)$ , it is known that the Riemann Hypothesis for  $\xi(s)$  implies the Riemann hypothesis for  $\xi^{(m)}(s)$ , where  $m$  is any positive integer. In this talk, we discuss the distribution of the fractional parts of the sequence  $(\alpha \gamma_m)$ , where  $\alpha$  is any fixed non-zero real number and  $\gamma_m$  runs over imaginary parts of zeros of  $\xi^{(m)}(s)$ . This involves a zero density estimate and an explicit formula for the zeros of  $\xi^{(m)}(s)$ . This is joint work with Arindam Roy.

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**ALEXANDER MANGEREL**, CRM, Université de Montréal

*The Variance of Bounded Multiplicative Functions in Arithmetic Progressions*

Let  $f : \mathbb{N} \rightarrow \mathbb{U}$  be a 1-bounded multiplicative function. Improving on work of Hooley, we establish unconditional (quantitative) asymptotic formulae for

$$\sum_{\substack{n \leq x \\ n \equiv a \pmod{q}}} f(n)$$

for *almost all* coprime residue classes  $a$  modulo  $q$ , and all but  $O\left(xe^{-(\log x)^{0.66}}\right)$  moduli  $q \in (x, 2x]$ . Time permitting, we will also discuss some conditional analogues (weaker than GRH) for which no exceptional set of moduli is required, as well as some extensions for divisor-bounded functions. (Joint work with O. Klurman and J. Teräväinen)

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**XIANCHENG MENG**, University of Göttingen  
*Chebyshev's bias for products of irreducible polynomials - I*

I will briefly introduce some of my work on the generalization of "Chebyshev's bias" to some restricted integers. Motivated by some ideas for dealing with the case of integers, joint with Lucile Devin, we consider the number of products of  $k$  irreducible polynomials over a finite field among different arithmetic progressions. We unconditionally obtain asymptotic formula for the difference of the counting functions uniformly for  $k$  in certain range. Then we derive the existence of the limiting distribution for the difference function. Due to the existence of possible central zeros of the associated  $L$ -functions, the difference function may behave very differently from the case of integers.

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**KYLE PRATT**, University of Illinois at Urbana-Champaign  
*Some non-vanishing results for Dirichlet  $L$ -functions*

There is a conjecture, going back in one form or other to Chowla, that the central values of Dirichlet  $L$ -functions are almost always non-zero. I will discuss some recent results on non-vanishing of Dirichlet  $L$ -functions at the central point (some of this work is joint with Siegfried Baluyot). I will mention some connections to conjectures of Keating-Snaith and Katz-Sarnak.

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**LOLA THOMPSON**, Oberlin College  
*Counting quaternion algebras*

We discuss how classical techniques from analytic number theory can be used to count quaternion algebras over number fields subject to various constraints. Because of the correspondence between maximal subfields of quaternion algebras and geodesics on arithmetic hyperbolic manifolds, these counts have interesting applications to the field of spectral geometry. This talk is based on a recent paper with Benjamin Linowitz, D. B. McReynolds, and Paul Pollack.

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**LEE TROUPE**, University of Lethbridge / PIMS  
*Distributions of polynomials of additive functions*

The celebrated Erdős–Kac theorem says that if an additive arithmetic function satisfies certain mild hypotheses, then its values obey a normal distribution. In the years since, the "Erdős–Kac class" of functions whose values are normally distributed has been broadened to include certain non-additive functions and arithmetic functions restricted to interesting subsets of the integers, such as shifted primes. This talk will focus on recent joint work with Greg Martin (UBC) that further expands the Erdős–Kac class to include arbitrary sums and products of additive functions satisfying Erdős and Kac's original requirements.

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**CAROLINE TURNAGE-BUTTERBAUGH**, Carleton College  
*A Chebotarev density theorem for families, and an application to class groups*

This talk will present an effective Chebotarev theorem that holds for all but a possible zero-density subfamily of certain families of number fields of fixed degree. For certain families, this work is unconditional, and in other cases it is conditional on the strong Artin conjecture and certain conjectures on counting number fields. As an application, we obtain nontrivial average

upper bounds on  $l$ -torsion in the class groups of the families of fields. This talk is on joint work with Lillian Pierce and Melanie Matchett Wood.

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**PENG-JIE WONG**, University of Lethbridge/PIMS

*A Bombieri-Vinogradov Theorem for Modular Forms*

For any  $(a, q) = 1$ , assuming the generalised Riemann hypothesis for Dirichlet  $L$ -functions, we have a strong form of Dirichlet's theorem on arithmetic progressions:

$$\pi(x, a, q) = \frac{1}{\phi(q)} \text{Li}(x) + O(x^{1/2} \log(qx)),$$

where  $\pi(x, a, q)$  stands for the number of primes  $p \leq x$  congruent to  $a$  modulo  $q$ ,  $\phi$  is Euler's totient function, and  $\text{Li}(x)$  is the logarithmic integral function. Nowadays, although the generalised Riemann hypothesis remains open, we still know that such an estimate is valid "on average" by the celebrated theorem of Bombieri and Vinogradov.

In this talk, we will consider a modular variant of both theorems that gives a count of Fourier coefficients of modular forms over arithmetic progressions. If time allows, we will also discuss some applications related to questions of Lehmer and Serre on the non-vanishing of Fourier coefficients of modular forms.

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**ASIF ZAMAN**, Stanford University

*Moments of other random multiplicative functions*

Random multiplicative functions naturally serve as models for oscillatory deterministic ones such as the Mobius function. After fixing a particular random model, there are many interesting questions one can ask. For example, what is the distribution of their partial sums? Harper has recently made remarkable progress for partial sums of certain random multiplicative functions with values that lie on the complex unit circle. He settled the correct order of magnitude of their  $q$ th moments for all real  $q \geq 0$  and surprisingly established that one expects better than square-root cancellation in their partial sums. I will discuss progress on extending Harper's analysis to a wider class of multiplicative functions such as those modeling the coefficients of higher degree  $L$ -functions.