AARMS-CMS Student Poster Session
Présentations par affiches des étudiants - AARMS-SMC
(Org: **Ismail Abouamal** (University of Toronto) and/et **Aaron Berk** (University of British Columbia))

EUGENE FILATOV, Simon Fraser University
Quaternion Algebras and the Burkhardt Quartic

The Burkhardt quartic threefold in $\mathbb{P}^4$ is given by

$$B : f(y_0, y_1, y_2, y_3, y_4) := y_0(y_0^3 + y_1^3 + y_2^3 + y_3^3 + y_4^3) + 3y_1y_2y_3y_4 = 0.$$

This variety has been studied extensively since 1890 (originally by Heinrich Burkhardt), and has several different characterizations. Points on the Burkhardt quartic correspond to the class of curves that admit a model of the form $y^2 = h(x)$ where h is a squarefree polynomial of degree 6, together with 40 decompositions of the form

$$h(x) = G(x)^2 + \lambda H(x)^3.$$

Part of this correspondence involves marking 6 points on a conic C, and in order to obtain 6 corresponding points on $\mathbb{P}^1$ for defining h, it is necessary that C has a k-rational point. The Burkhardt has another, natural symmetric model $B' \subset \mathbb{P}^5$ given by

$$B' : \sigma_1(y_0, ..., y_5) = \sigma_4(y_0, ..., y_5) = 0,$$

where the σ_i are elementary symmetric functions. This model and the original Burkhardt are isomorphic over $\mathbb{C}$ (in fact over $\mathbb{Q}(\zeta_3)$), so they are geometrically equivalent. However, they are not isomorphic over $\mathbb{Q}$. In other words, B' is a nontrivial twist of B. Several properties over $\mathbb{Q}$ change drastically upon twisting the Burkhardt, in particular whether or not the conic C has $\mathbb{Q}$-rational points (for instance when obtained from B it does, while from B' there are local obstructions over $\mathbb{R}$ and $\mathbb{Q}_3$).

JABAR S. HASSAN, Missouri University of Science and Technology
New RKHSs on a semi-infinite domain with existence and uniqueness results for the non-homogeneous telegraph equation

In this talk we introduce new reproducing kernel Hilbert spaces(RKHSs) on a semi-infinite domain and demonstrate existence and uniqueness of solutions to a certain type of non-homogeneous hyperbolic partial differential equation in these spaces if the driver is square integrable and sufficiently smooth. These spaces are reasonable for applications and convenient for numerical approximation because the kernels are piece-wise polynomial functions.

JABAR SALIH HASSAN, Missouri University of Science and Technology

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ROBYN HEARN, Simon Fraser University
Counting Irreducible Polynomials over $\mathbb{Z}_n$

It is commonly known that there are $\binom{n}{2}$ irreducible, quadratic polynomials over $\mathbb{Z}_n$ when n is a prime. Then it is natural to ask how many irreducible quadratics there are over $\mathbb{Z}_n$ without the condition that n is prime. Counting methods for the case where n is prime, rely on field axioms which $\mathbb{Z}_n$ does not generally satisfy. In this thesis, we relate counting reducible, quadratic polynomials to the simpler problem of counting squares. In the construction of this count, we find an algorithm to generate all quadratic polynomials over $\mathbb{Z}_n$ and categorize them as reducible or irreducible. This algorithm is computationally less expensive than the naive cubic algorithm for generating all irreducible quadratics.

DARRICK LEE, University of Pennsylvania
Path Signatures and Topological Time Series Analysis

Let $\{\gamma_i\}_{i=1}^n$ be a collection of n simultaneous time series $\gamma_i : [0,1] \to \mathbb{R}$. By viewing this collection of time series as a path $\gamma = (\gamma_1, \ldots, \gamma_n) \in P\mathbb{R}^n$, we may consider the 0-cochains in Chen's iterated integral cochain model for $P\mathbb{R}^n$ as a complete, reparametrization-invariant feature set for paths (and thus collections of time series). Standard tools for time series analysis such as Fourier analysis and cross-correlation often depend on the exact parametrization of the input, but measured data is rarely parametrized in the same manner as the underlying value of interest. In addition, many properties of interest, such as the existence of a lead-lag relationship between time series, are also not parametrization dependent. We present algebraic and statistical properties of path signatures, and outline possible applications.

RICHARD LEYLAND, Queen's University
Galois Representations of CM Elliptic Curves

In our research we study to the following problem inspired by Mazur: Let E be an elliptic curve with complex multiplication defined over a number field F and let N be a positive integer. We wish to determine all elliptic curves E'/F with $\mathrm{Gal}(\bar{F}/F)$-isomorphisms $E[N] \cong E'[N]$. In addition, we ask if there are any $\mathrm{Gal}(\bar{F}/F)$-isomorphisms that are not induced by F-rational isogenies. In this poster we review some of the known results regarding isogenies of CM elliptic curves and Galois representations of CM elliptic curves.

YUE LIU, University of British Columbia
Analysis of reaction-diffusion PDE models for localized pattern formation in cells

We study a system of reaction-diffusion PDEs, which models the dynamics of small GTPases, a family of signalling proteins. The system is composed of 3 fully coupled quantities, a fast-diffusing active protein, a slow-diffusing inactive form (with nonlinear interconversion term) and slow negative feedback from a and a non-diffusive quantity (F-actin). We explore the system's bifurcation behaviors using local perturbation analysis (a shortcut to probe the growth/decay of localized spikes), and look for spatio-temporal patterns using numerical simulations. The results have biological applications toward understanding cell motility phenomena.

CALEB MARSHALL, Missouri State University
Chains of Dot Products over Finite Point Sets in $\mathbb{R}^2$

Given a finite point set $\Lambda \subset \mathbb{R}^2$, we examine the behavior of successive dot products between sequences of distinct points. Fix $k \in \mathbb{N}$. Now, let $\{\alpha_i\}_{i=1}^k$ be a sequence of numbers, $0 < \alpha_i < 1$, and let $\{R_i\}_{i=1}^{k+1}$ be a sequence of points such that $R_i \cdot R_{i+1} = \alpha_i$ for each $1 \leq i \leq k+1$. Then, together, we call the sequences $\{\alpha_i\}_{i=1}^k$, $\{R_i\}_{i=1}^{k+1}$ a *k-chain*. Now, given that $\#\Lambda = N$, we prove that, for fixed k, the maximum number of k-chains that can exist over Λ is $N^{\frac{4}{3}(\lceil \frac{k+1}{2} \rceil)}$. We also construct a point-set which sharpens this bound in $\mathbb{R}^2$. Finally, we explore extensions of this result, and its motivations in questions of Euclidean distances originally proposed by Paul Erdős.

MASOOD MASJOODY, Simon Fraser University
The Game of Cops and Robber on Graphs with Two Forbidden Induced Subgraphs

It is known that the class of all graphs not containing a connected graph H as an induced subgraph is cop-bounded if and only if H is a path. In this study we show that for a set $\mathcal{H} = \{H_1, H_2\}$ of two non-path connected graphs, the class of $\mathcal{H}$-free graphs, i.e. the class of all graphs not containing an element of $\mathcal{H}$ as an induced subgraph, is cop-bounded only if $\mathcal{H}$ consists of a generalized claw and a generalized net. For the reverse direction, we provide upper bounds for the cop-number of $\mathcal{H}$-free graphs for the following particular cases (1) when H_1 is the $(1,1,a)$-claw and H_2 is the $(1,1,b)$-net, and (2) when H_1 is the $(1,a,b)$-claw and H_2 is the $(0,b,c)$-net, where a, b, c are constant positive integers.

DANIEL MESSENGER, Simon Fraser University (SFU)
Random Interacting Particle Systems: Numerics for the Zero-Diffusion Limit

Collective behavior of large systems of interacting agents, or swarming, can be described mathematically using a general framework which applies to situations as various as flocks of birds and opinion dynamics. We investigate swarming in domains with boundaries and quantify the effect of adding linear diffusion to the model, or Brownian motion at the particle level. Using a Monte-Carlo sampling algorithm combined with particle simulations, we show numerically that in the zero-diffusion limit, solutions to the aggregation-diffusion equation converge to solutions of the plain (non-diffusive) aggregation model at the predicted analytical rate.

NICOLA MULBERRY, Simon Fraser University
HIV Epidemic Control among Sex Workers and their Clients

Controlling the spread of HIV among hidden, high-risk populations such as sex workers and their clients is becoming increasingly important in the fight to end AIDS. We identify a number of sociological and structural factors which render general control strategies ineffective among these key populations, and instead call for focused testing and interventions. A bipartite network model of sexual contacts between female sex workers and male clients is motivated using historical data from a South African mining community. HIV transmission and progression is modelled as a stochastic process on the network, and the effect of various intervention strategies on HIV prevalence in the population is determined through numerical simulations. We find that preventative interventions are highly cost-effective when targeted at female sex workers. For aggressive reduction in HIV prevalence, however, the client population cannot be ignored and treatment of both populations is necessary.

FARZANEH NIKBAKHTSARVESTANI, University of Manitoba
A CHARACTERIZATION OF THE GENERALIZED KKM MAPPINGS VIA THE MEASURE OF NONCOMPACTNESS IN COMPLETE GEODESIC SPACES

By using the measure of noncompactness, we characterize the class of generalized KKM mappings on complete geodesic spaces. Existence theorems of fixed points and the best approximation for set-valued mappings are established. Applications are also provided. The results presented in this paper improve and extend some recent results in the literature.

GARRETT PALUCK, Simon Fraser University
Hensel Lifting Dense Bivariate Polynomials

We present a new algorithm for performing linear Hensel Lifting of bivariate polynomials over a finite field. The sequential version of our algorithm has a running of $O(nd^3)$ for lifting n univariate polynomials of degree d with respect to a bivariate polynomial of degree d in both variables, assuming that we are using classical polynomial multiplication.

WESTON CHRISTOPHER RODA, University of Alberta
Effectiveness of shock and kill strategy for eliminating HIV-1 brain infection: a mathematical modeling study

Antiretroviral therapy (ART) has greatly reduced the overall morbidity and mortality of human immunodeficiency virus-1 (HIV-1) infected patients. Even with the success of ART, the virus persists in many different cells and tissues, and tissues that have minimal ART penetration and limited host immune responses make ideal locations for viral reservoirs. These viral reservoirs contain latently infected long-lived cells. The "Shock and Kill" therapy aims to reactivate latently infected cells by latency reversing agents (LRAs) and kill these reactivated cells by strategies involving the host immune system and certain drugs. The brain is a natural anatomical reservoir for HIV-1 infection. A mathematical model was used to qualitatively analyze the dynamics of latently and productively infected cells in the brain during viral infection and this mathematical model was used to simulate the "Shock and Kill" therapy in the brain. Our model produces the clinical and experimental observation that effective ART can suppress productively infected brain macrophages but leaves a latent reservoir of brain macrophages. By

adding a reactivation rate of the latent reservoir into the model, the "Shock and Kill" therapy is analyzed both mathematically and numerically to investigate strategies leading to the eradication of the latent reservoir of brain macrophages.

ARSHDEEP SINGH SANDHU, University of British Columbia

Tracking Spots During Pattern Formation On A Surface

The beautiful spotted coats of leopards and cheetahs pose an interesting question: how do such patterns form in nature? Many researchers use mathematical models based on reaction-diffusion differential equations to study these sorts of problems.

Our project involves tracking moving spots in a pattern as they evolve on a sphere. The spots are the result of numerical simulations of the Brusselator reaction-diffusion system. We use clustering algorithms from machine learning to extract the location of the spots from a large point cloud of time-dependent data. We then create traces showing the paths taken by the spots as they evolve.

Eventually, we hope to use our algorithm to help study the dynamics of splitting spots on a growing domain where spots are competing for space.

HYUNGEUN SHIN, University of Victoria

Effect of tropical waves on the tropical cyclone drift

The prediction of tropical cyclone motion and path have been for a long time a major topic in geophysical science and weather forecasting. The bulk of the motion and path of a tropical cyclone are determined by the so-called beta-gyres that form on its sides due to the variation in the Coriolis parameter with latitude–the so-called beta-drift. Despite such important progress in our understanding, the motion of tropical cyclones is not easily estimated because of dynamical interactions with ambient waves and the background flow. Thus, researchers have investigated how the motion of a tropical cyclone is affected by the environmental flows using numerical simulations. In this study, various types of equatorial waves are considered as the environmental flows for northern hemisphere tropical cyclones. The motion of the tropical cyclone is simulated using a 2D barotropic model. The domain of simulation is from 45 degree north to 45 degree south and all around the earth. For the boundary conditions, east-west is symmetric, and there is no y-direction flow at north-south boundaries. As a consequence, it is common that cyclones move northwestward, but the movement angle and traveling distance of cyclones are different depending on the types of equatorial waves. In addition, the differences become more significant after 12h because the waves have different influences on the beta gyres.

MOHAMMAD SHIRAZI, University of Manitoba

Grunsky Operator and Period Mappings for Surfaces with One Border

Consider a Riemann surface $\mathbf{R}$ with one border which can be described as a compact Riemann surface with a domain, say Ω, removed from it. The domain Ω can be considered as the image of the unit disc **D** in the complex plane, under a conformal map f with quasi-conformal extension. Let Γ be the boundary of Ω, positively oriented with respect to Ω, and Σ the complement of $cl\Omega$ in R.

We aim to characterize the function space $\mathcal{D}(\Sigma)$, the set of all the holomorphic functions defined on Σ of bounded Dirichlet norm, and the set of its boundary values on Γ.

In this poster, we will also characterize all the functions $\mathcal{D}_{harm}(\Sigma)$ that carry the jump decomposition. We then introduce an operator I_f, called Faber operator, between the space $\mathcal{D}(\Sigma)$ and a quotient space of $\overline{\mathcal{D}(\mathbf{D})}$ (the set of all anti-holomorphic functions on **D** of bounded Dirichlet norm) and show that under what circumstances I_f is an isomorphism. The Faber operator generalizes the Faber operator of approximation theory of holomorphic functions on planar domains to Riemann surfaces. We will construct another operator Gr_f, called Grunsky operator, which can be interpreted as a generalization of a period matrix of a genus g compact Riemann surface. The Dirichlet space is then characterized as the graph of this operator.

Finally, the connection between anti-holomorphic $1-$forms on Ω and holomorphic $1-$forms on Σ and the important role of Schiffer operator will also be discussed.

Joint work with E. Schippers and W. Staubach

STEFAN TRANDAFIR, Simon Fraser University
An analytic approach to computing Kronecker coefficients

Kronecker coefficients appear as structure constants for the decomposition into irreducibles of the tensor product of representations of the symmetric group. Since the 1938 article 'The Analysis of the Kronecker Product of Irreducible Representations of the Symmetric Group' written by Murnaghan, there has been an active interest in finding a combinatorial interpretation of the Kronecker coefficients. Unlike the closely related Littlewood-Richardson coefficients, which count points in polytopes, the Kronecker coefficients have no known combinatorial interpretation, and cannot be feasibly computed with existing methods. It is even unkown when they are non-zero.

This work builds upon advances made by Baldoni, Vergne, and Walter (2016) who showed that, for fixed parameter sizes, the Kronecker coefficients are piecewise quasi-polynomials. They also provided theoretical computation methods, which are unfortunately too complex to be implemented in practice. More recently, Mishna, Rosas and Sundaram have found a way to reduce the complexity by linking the coefficients more directly to the number of integer points in specific polyhedra. This interpretation opens the door towards a complex analytic approach. We utilise techniques from the relatively new field of multivariate analytic combinatorics (described in 'Analytic Combinatorics in Several Variables' by Pemantle and Wilson, 2013) in order to determine asymptotic formulas for the Kronecker coefficients. Such formulas yield important information about the coefficients, notably, determining where the Kronecker coefficients are 0.

RAYMOND WALSH, Simon Fraser University
A Dynamical Theory for Cloud Edge Motion

Much is known about cloud formation and their behaviour at large scales (tens of kilometers). Considerably less, in atmospheric science, addresses the fluid mechanics dictating smaller scale motions (tens of meters) that determine the shapes of cloud edges. An example of this is the formation of a holepunch cloud. A curious phenomena whereby a growing circular hole in a shallow cloud layer opens up due to a disturbance typically initiated via aircraft. Only recently (2015) has the mechanism for its formation been understood which stands as the motivation for this work.

We present a new model for the edge motion of stable non-precipitating clouds. Our proposed model combines buoyancy driven fluid mechanics and the thermodynamic theory for phase change of water into a self-consistent Boussinesq theory. Further approximation including linearization of the thermodynamics about a cloud forming state and leveraging the low water content of the atmosphere leads to an analytically tractable free boundary model for cloud edge motion. Exact solutions to this model then serve as a theoretical benchmark against which full physics numerical models can be compared. In particular, we present results from the cm1 model (cloud model 1) developed by George Bryan out of the National Center for Atmospheric Research in Boulder Colorado. We finish by presenting an analytically calculated travelling wave solution on the clear/cloud interface for which the dispersion relation is derived and compared.

WEIJIA YIN, University of British Columbia
Improved Hardy Inequalities with Exact Remainder Terms

Hardy inequalities play extremely important roles in analysis, probability and partial differential equations.We set up several identities that imply some versions of the Hardy type inequalities.These inequalities give a straightforward understanding of several Hardy type inequalities as well as the nonexistence of nontrivial optimizers.These identities include adding the exact terms of Hardy inequalities and its improved version on bounded domains and the exact reminders of improved versions of Hardy-inequalities with N-dimensional Bessel pair on $(0, R)$

ALEXANDRE ZOTINE, Simon Fraser University
Global Generation of Vector Bundles over Elliptic Curves

Given an algebraic set X (e.g. a curve or surface), a vector bundle over X associates a vector space to each point of X in a smooth manner. In 1957, Sir Michael Atiyah provided a classification of vector bundles over elliptic curves. Using this

classification, the goal of our research is to determine when a vector bundle over an elliptic curve is globally generated. This poster is meant to introduce preliminaries to the problem, as well as present some progress towards its resolution.