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Grunsky Operator and Period Mappings for Surfaces with One Border

Consider a Riemann surface $\mathbf{R}$ with one border which can be described as a compact Riemann surface with a domain, say Ω, removed from it. The domain Ω can be considered as the image of the unit disc **D** in the complex plane, under a conformal map f with quasi-conformal extension. Let Γ be the boundary of Ω, positively oriented with respect to Ω, and Σ the complement of $cl\Omega$ in R.

We aim to characterize the function space $\mathcal{D}(\Sigma)$, the set of all the holomorphic functions defined on Σ of bounded Dirichlet norm, and the set of its boundary values on Γ.

In this poster, we will also characterize all the functions $\mathcal{D}_{harm}(\Sigma)$ that carry the jump decomposition. We then introduce an operator I_f, called Faber operator, between the space $\mathcal{D}(\Sigma)$ and a quotient space of $\overline{\mathcal{D}(\mathbf{D})}$ (the set of all anti-holomorphic functions on **D** of bounded Dirichlet norm) and show that under what circumstances I_f is an isomorphism. The Faber operator generalizes the Faber operator of approximation theory of holomorphic functions on planar domains to Riemann surfaces. We will construct another operator Gr_f, called Grunsky operator, which can be interpreted as a generalization of a period matrix of a genus g compact Riemann surface. The Dirichlet space is then characterized as the graph of this operator.

Finally, the connection between anti-holomorphic $1-$forms on Ω and holomorphic $1-$forms on Σ and the important role of Schiffer operator will also be discussed.

Joint work with E. Schippers and W. Staubach