Symbolic and Regular Powers of Ideals
Puissances symboliques et régulières des idéaux

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ALI ALILOOEE, Bradley University
Packing properties of some classes of cubic squarefree monomial ideals

Let I be an ideal in a Noetherian ring R. Its n-th symbolic power of I is defined as

$$I^{(n)} = \bigcap_{p \in Ass(R)} (I^n R_p \cap R).$$

Symbolic powers for several classes of ideals have been investigated for many years. The symbolic powers in general are not equal to the ordinary powers. Therefore, one interesting question here is for what classes of ideals ordinary and symbolic powers coincide? The answer for this question for squarefree monomial ideals may be packing property. In this talk we will briefly survey packing property for squarefree monomial ideals from combinatorial and algebraic aspects. Then we will focus on the cubic squarefree monomial ideals and we will see some new results.

ALESSANDRO DE STEFANI, University of Nebraska, Lincoln
Symbolic powers in mixed characteristic

The Zariski-Nagata Theorem in one of its classical versions states that, if P is a prime ideal in a polynomial ring over the complex numbers, then the n-th symbolic power of P consists of all the polynomial functions that vanish to order at least n along the variety defined by P. We present analogous results in mixed characteristic, combining properties of differential operators and p-derivations. This talk is based on joint work with Eloísa Grifo and Jack Jeffries.

BENJAMIN DRABKIN, University of Nebraska - Lincoln
Symbolic Defect and Cover Ideals

Let R be a commutative Noetherian ring, and let I be an ideal in R. The symbolic defect is a numerical measurement of the difference between the symbolic and ordinary powers of I. In the case that I has sufficiently well-behaved symbolic powers (i.e. its symbolic Rees algebra is finitely generated) we prove that the symbolic defect grows eventually quasi-polynomially. Furthermore, we describe more specifically the growth of the symbolic defect in certain classes of ideals arising from graphs, termed cover ideals.

CHRIS FRANCISCO, Oklahoma State University
Some new results on asymptotic resurgence

We give some new results on asymptotic resurgence, an invariant introduced by Guardo, Harbourne, and Van Tuyl. Our focus will be on the monomial case and will incorporate earlier work of Cooper, Embree, Hà, and Hoefel.

FEDERICO GALETTO, Cleveland State University
Betti numbers of symbolic powers of star configurations

At the 2017 CMO-BIRS workshop on Ordinary and Symbolic Powers of Ideals, a project started to compute Betti numbers for symbolic powers of the defining ideals of star configurations in projective space. I will provide an update on the status of the project. In particular, I will introduce a more general class of ideals connected to the original problem. Our work (in progress) shows these ideals have linear quotients thus providing a technique for computing their Betti numbers.

ELOÍSA GRIFO, University of Michigan
A stable version of Harbourne's Conjecture

Given a radical ideal I of big height h in a regular ring R, Harbourne conjectured that the famous containment result by Ein–Lazersfeld–Smith, Hochster–Huneke and most recently Ma–Schwede could be improved to $I^{(hn-h+1)} \subseteq I^n$. Unfortunately, several counterexamples have been found for specific values of n. In this talk, we will discuss evidence pointing to a possible stable version of Harbourne's Conjecture for all ideals I: that $I^{(hn-h+1)} \subseteq I^n$ might hold for all sufficiently large values of n.

ELENA GUARDO, Università di Catania
On the Waldschmidt constant and resurgence of some ideals in $\mathbb{P}^n$ and $\mathbb{P}^1 \times \mathbb{P}^1$

We are interested in the ideal containment problem: given a nontrivial homogeneous ideal I of a polynomial ring $R = k[x_1, \ldots, x_n]$ over a field k, the problem is to determine all positive integer pairs (m, r) such that $I^{(m)} \subseteq I^r$. Most of the work done up to now has been done for ideals defining 0-dimensional subschemes of projective space. Here, we focus on certain ideals defined by a union of lines in $\mathbb{P}^3$ which can also be viewed as points in $\mathbb{P}^1 \times \mathbb{P}^1$. We also consider ideals of s general lines in $\mathbb{P}^n$. We give results in the case of squarefree monomial ideals. This talk is based on joint papers with B. Harbourne, A. Van Tuyl and MFO Group.

BRIAN HARBOURNE, University of Nebraska-Lincoln
Primer on Unexpected Varieties

In this talk I will add texture to the overview provided in the talk by J. Migliore by giving specific examples which will be useful for anyone considering to begin doing research on this topic. Along the way I will highlight what is known and what is still open regarding this topic.

MARYAM EHYA JAHROMI, Dalhousie
Stability of Powers of Cover Ideals of Hypergraphs

We introduce a family of hypergraphs and study the associated primes of their cover ideals and their stability.

This family of hypergraphs provides an answer to the question arisen in 2010 by C. Francisco, T. Há, and A. Van Tuyl in their paper "Colorings of Hypergraphs, Perfect Graphs, and Associated Primes of Powers of Monomial Ideals".

Does there exist a family of hypergraphs $\{\mathcal{H}_n\}_{n\in\mathbb{N}}$ such that

$$\text{astab}(J(\mathcal{H}_n)) \geq (\chi(\mathcal{H}_n) - 1) + n$$

For all $n \in \mathbb{N}$?

Later in 2013, A. Bhat, J. Biermann, and A. Van Tuyl introduced a family of hypergraphs which satisfies the inequality. In our work we find a new family of hypergraphs for which we can replace the inequality by equality.

JUAN MIGLIORE, University of Notre Dame
Unexpected hypersurfaces

In the last half decade or so there has been a growing amount of work on the topic of configurations of points in projective space that admit "unexpected" curves or hypersurfaces. This began in the setting of the projective plane, but more recently has moved in interesting ways into the more general setting of higher dimensional projective space. There have been surprising connections to such diverse things as rank 2 vector bundles on the plane, line arrangements in the plane, Terao's conjecture, the Weak and Strong Lefschetz Properties, hyperplane arrangements, and root systems. Also, interesting geometric properties of our configurations have recently presented themselves. I will begin an overview of this work, with emphasis on a recent preprint of B. Harbourne, U. Nagel, Z. Teitler and myself. Brian Harbourne will continue in his talk, describing other aspects of this joint work.

SUSAN MOREY, Texas State University

Depth and Regularity of Regular and Symbolic Powers of Monomial Ideals

This talk will cover recent results related to the depth and regularity of general monomial ideals with a goal of applying the results to determine related properties of regular and symbolic powers of square-free monomial ideals. In recent work with Jose Martinez-Bernal, Rafael Villarreal, and Carlos Vivares, polarization was used to examine how altering the powers of variables in monomial generators affects the depth and regularity of the ideal. As an application, we are able to identify classes of square-free monomial ideals for which the powers, or the symbolic powers, have non-increasing depth and non-decreasing regularity.

ALEXANDRA SECELEANU, University of Nebraska-Lincoln

Minimum distance functions for fat point ideals

In coding theory, a code is a linear subspace of a finite dimensional vector space. The Hamming distance of the code is the minimum number of nonzero entries in a nonzero element (codeword) of this subspace. Hamming distance has a nice geometric interpretation: if the elements of a basis for the code are viewed as coordinates for a set of points in projective space and if these points are distinct, then the Hamming distance can be computed bases on the maximum number among these points that are contained in a hyperplane.

Motivated by these considerations, we present generalizations for the notion of Hamming distance for fat points and, more generally, for arbitrary homogeneous ideals. This family of numerical functions is termed minimum distance functions. This talk is based on joint work with Susan Cooper, Stefan Tohaneanu, Maria Vaz Pinto and Rafael Villarreal.

TOMASZ SZEMBERG, Pedagogical University of Cracow

Unexpected algebraic and geometric properties of Fermat-type configurations.

For a positive integer n, a Fermat arrangement of lines in $\mathbb{P}^2$ is given by linear factors of the polynomial

$$F_{2,n} = (x^n - y^n)(y^n - z^n)(z^n - x^n).$$

There is an associated configuration of points $Z_{2,n}$ determined by an almost complete intersection ideal

$$I_{2,n} = (x(y^n - z^n), y(z^n - x^n), z(x^n - y^n)).$$

These configurations provide interesting examples for the Containment Problem and in the area of unexpected hypersurfaces.

In my talk, based on a work in progress with Justyna Szpond, I will report on several interesting generalizations to higher dimensional spaces.

WILLIAM TROK, University of Kentucky

Hyperplane Arrangements and Extremal Behavior

Many interesting hyperplane arrangements have been studied recently by commutative algebraists, as they have been shown to be related to objects with interesting properties. Particularly, ideals failing the containment $I^{(3)} \subseteq I^2$, and sets of points whose ideal has larger than expected intersection with a power of a generic point ideal. We discuss some of these examples, and some results showing how certain properties of the Hyperplane Arrangements may play a role in the above phenomenon.