In this talk, we study well-posedness of the Cauchy problem to the forth-order nonlinear Schrödinger equations with \( \gamma \in \{1, 2, 3\}\)-times derivative nonlinearities in Sobolev space \( \mathcal{H}^s(\mathbb{R}) \):

\[
\begin{cases}
  i\partial_t u + \partial^4_x u = G \left( (\partial^k_x u)_{k\leq \gamma}, (\partial^k_x \bar{u})_{k\leq \gamma} \right), \quad (t, x) \in I \times \mathbb{R}, \\
  u|_{t=0} = u_0 \in \mathcal{H}^s(\mathbb{R}),
\end{cases}
\]

where \( u : I \times \mathbb{R} \to \mathbb{C} \) is an unknown function, \( I := [-T, T] \) denotes the existence time interval of the function \( u, u_0 \in \mathcal{H}^s(\mathbb{R}) \) is a prescribed function, and for \( s \in \mathbb{R}, \mathcal{H}^s(\mathbb{R}) \) denotes \( L^2(\mathbb{R}) \)-based Sobolev space. For \( m \in \mathbb{N} \) with \( m \geq 3 \), we mainly consider the \( m \)-th order nonlinearity \( G \) of the form

\[
G(z) = G(z_1, \cdots, z_{2(\gamma+1)}) := \sum_{|\alpha|=m} C_\alpha z^\alpha,
\]

where \( C_\alpha \in \mathbb{C} \) with \( \alpha \in (\mathbb{N} \cup \{0\})^{2(\gamma+1)} \) are constants. The purpose of this talk is to improve the previous results obtained by several Mathematicians, that is, to treat more general nonlinearity and to prove local well-posedness of the problem in lower order Sobolev space \( \mathcal{H}^s(\mathbb{R}) \). Our proof of the well-posedness result is based on the contraction argument on a suitable function space, via the Strichartz estimates, Kato-type smoothing estimates, Kenig-Ruiz estimates, Maximal function estimates, a linear estimate for inhomogeneous term, the bilinear Strichartz type estimate and the Littlewood-Paley theory.