As a simplified model derived from the Navier-Stokes equations, we consider the viscous Burgers equations in $\mathbb{R}$,

$$u_\epsilon^t - \epsilon u_{xx} + \frac{(u_\epsilon^t)^2}{2} = f(x,t), \quad x \in \mathbb{R}, \quad t \geq 0$$

$$u_\epsilon^t(x,0) = u_0(x), \quad x \in \mathbb{R},$$

$$u_\epsilon^t \to g \text{ as } x \to -\infty, \quad u_\epsilon^t \to h \text{ as } x \to \infty \text{ and } g > 0 > h, \quad \forall t \geq 0.$$  \hspace{1cm} (1)

We investigate the singular behaviors of their solutions $u_\epsilon^t$ as the viscosity parameter $\epsilon$ gets smaller. Indeed, when $\epsilon$ gets smaller, $u_\epsilon^t$ has viscous shocks whose slopes are proportional to $1/\epsilon$. So controlling the sharp layer is one of the most important parts in this research.

The idea is constructing the asymptotic expansions in the order of the $\epsilon$ and validating the convergence of the expansions to the solutions $u^t$ as $\epsilon \to 0$ in $L^2(0,T;H^1(\mathbb{R}))$ space. In this article, we consider the case where a single viscous shock occurs, i.e. interior layers, and we fully analyse the convergence at any order of $\epsilon$ using the so-called interior layer correctors.