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Solving S-unit equations in Sage and Applications

Let K be a number field. Many finiteness results in number theory and arithmetic geometry rely on the fact, due to Thue, Siegel, Mahler, and Lang, that for any non-zero a and b in K, the equation ax+by=1 has only finitely many solutions in any finitely generated multiplicative subgroup of K. A particularly useful consequence of this result is that the equation x+y=1 has finitely many solutions in the S-units of K. Work of Baker, Yu, de Weger, Smart, and many others resulted in practical algorithms for determining these solutions. However, until now, there has been no publicly available implementation of these algorithms in a computer algebra system. A group consisting of Alejandra Alvarado, Angelos Koutsianas, me, Chris Rasmussen, Christelle Vincent, and Mckenzie West has recently implemented functions in Sage to solve the S-unit equation for general K and S. In this talk, I will outline the algorithms, discuss current computational limitations, and share a few applications (and potential applications) in algebraic curves and number theory.