A quasigroup \((Q, \cdot)\) is an algebraic structure whose multiplication table is a Latin square. We say that \((x, y, z) \in Q^3\) is an associative triple if \((x \cdot y) \cdot z = x \cdot (y \cdot z)\). Quasigroups with few associative triples were proposed for various applications in cryptography. Let \(a(Q)\) denote the number of associative triples in \(Q\). It is easy to show that \(a(Q) \geq |Q|\). In 2012 Grošek and Horák conjectured that \(a(Q) = |Q|\) never occurs. Let us call \(Q\) maximally non-associative if \(a(Q) = |Q|\). The first example of a maximally non-associative quasigroup (of order 9) was found by Drápal and Valent (J. Combin. Des. 2018). In this work we use nearfields and their associated sharply two-transitive groups to construct maximally non-associative quasigroups. We conjecture that any nearfield that is not a field produces examples. We report results of an extensive and successful computer search. When \(q\) is an odd prime power, we show that a non-constructive existence result for maximally non-associative quasigroups of order \(q^2\) can be obtained if certain character sums can be suitably bounded. This is joint work with Aleš Drápal (Charles University, Prague).